Windows Connections

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Preliminaries

The approach in this presentation
- Take aways
  - Window types
  - Window relationships
  - Windows tables of information
  - Windows references

What are windows?
- Real, Finite, Possess DFT

How do we look at windows?
- Literally: DFT
- Figuratively: Parameters

Window Parameters

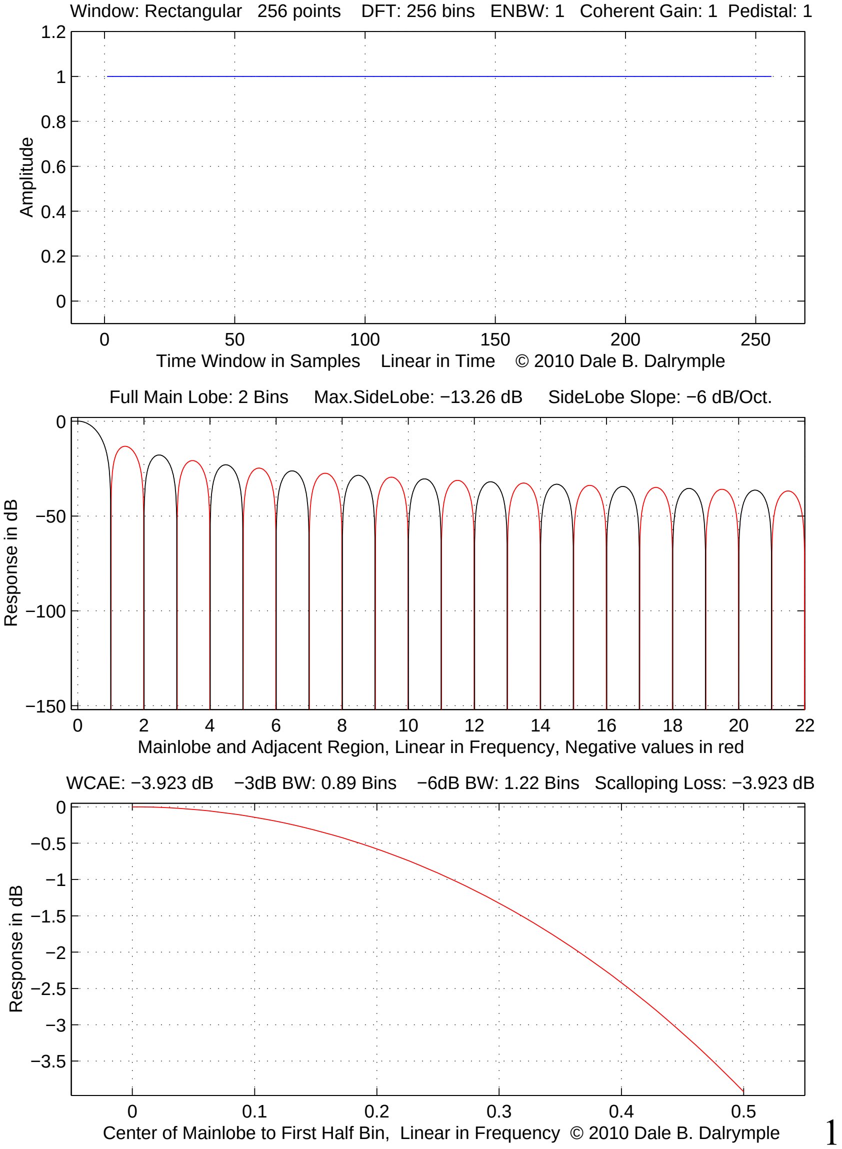
Parameter	**Units**	**Definition**	**References**
Coherent Gain	ratio	sum of normalized window divided by window length	harris
ENBW Effective Noise Bandwidth	bins	sum of the squares of the window coefficients divided by the square of the sum	harris
Scalloping Loss	dB	minimum possible value of the maximum gain of the adjacent bins for a tone not on bin center	harris
WCPL Worst Case Processing Loss	dB	scalloping loss minus the ENBW (in dB)	harris
-3dB Bandwidth	bins	bandwidth containing highest frequency response exceeding -3dB	harris
-6dB Bandwidth	bins	bandwidth containing highest frequency response exceeding -6dB	harris
-60dB Bandwidth	bins	bandwidth containing highest frequency response exceeding -60dB	
1st Zero Crossing	bins	frequency of first zero crossing defining the edge of the mainlobe	
1st Sidelobe Level	dB re: DC	response at peak of first sidelobe	
1st Sidelobe Frequency	fraction of Fs	frequency of peak of first sidelobe	
MSL Maximum Sidelobe Level	dB re: DC	harris “highest sidelobe level”	harris
Maximum Sidelobe Bandwidth	fraction of Fs	width of main lobe at level of MSL, parameter “b” in G&Y	G&Y
SLRO Sidelobe rolloff	dB/octave	harris “sidelobe falloff” minus the parameter “d” in	harris G&Y

Parameter	**Units**	**Definition**	**References**
		G&Y	
Bin 1 Intercept	dB	similar to “a2” in G&Y, altered to bin 1 using SLRO	G&Y
Window Pedistal		value at first sample, even FFT	
-3dB Efficiency	fractional (%/100)	fraction of total energy of response within -3dB bandwidth	
-6dB Efficiency	fractional (%/100)	fraction of total energy of response within -6dB bandwidth	
MRE Mainlobe Remaining Energy	fractional (%/100)	fraction of energy outside of mainlobe	

Gecklili, N. C., and Yarns, D., *"Some Novel Windows and a Concise Tutorial Comparison of Window Families"*, IEEE Trans. Acoust. Speech Sig. Proc, ASSP-26, pp. 501-507, Dec. 1978.

harris, f.j., *"On the Use of Windows for Harmonic Analysis with the Discrete Fourier Transform*” Proc. IEEE, 66, pp. 51-83, January 1978.
http://web.mit.edu/xiphmont/Public/windows.pdf
A portion of this is available (20100404) at:
http://www.signumconcepts.com/download/paper001.pdf

Rectangular Window Figure 1



This is what you get with “no window”.

Top plot:

Time domain window

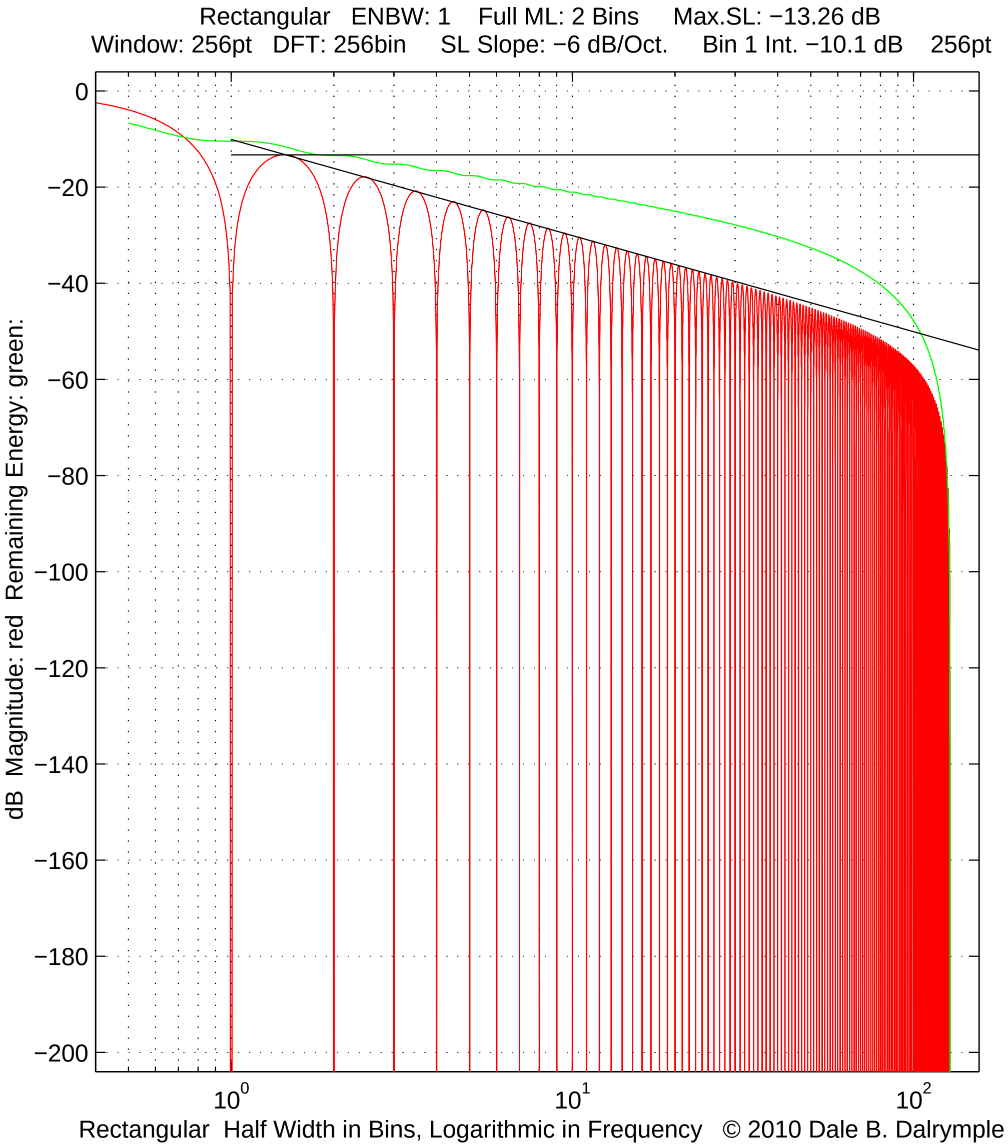
Middle plot:

Frequency domain plot (linear in frequency): the sinc function. Zeros at integer offsets from bin center. This is convenient for the most common DFT definition.

Bottom plot:

Bin top response, bin center to mid-point between bins shows scalloping loss.

Rectangular Window Frequency Response   Figure 2



<u>This is what you get with “no window”.</u>

This is a sinc function with a mainlobe width of 2 bins and zeros at (non-zero) integer offsets from zero frequency

Cosine Sum Windows

von Hann

Maximim sidelobe rolloff

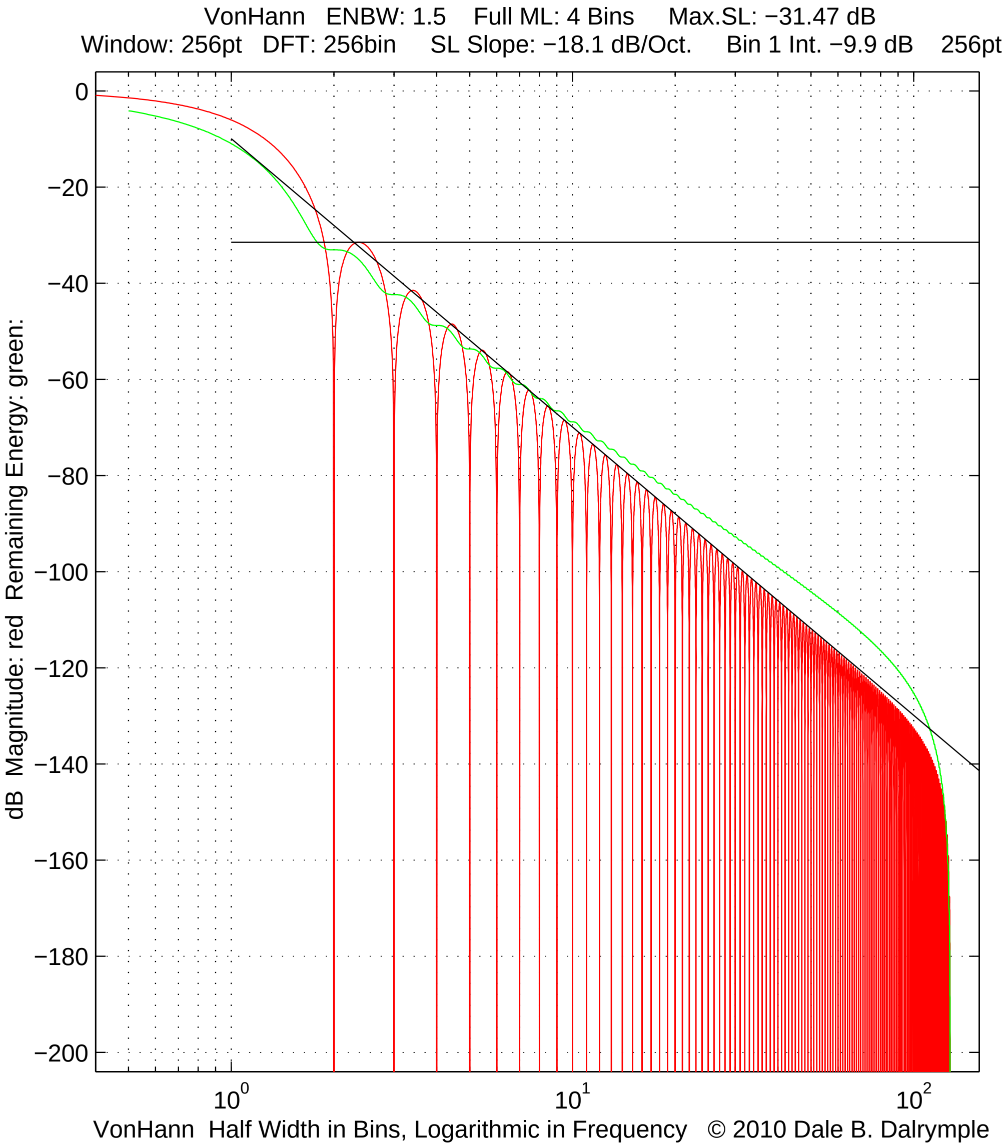
	Odd	Even
2	[1 1]/2 => -0.25 0.5 -0.25 in Freq Dom	[3 1]/4 =>-0.125 0.375 0.375 -0.125 inFreqDom
3	[3 4 1]/8	[10 5 1]/16
4	[10 15 6 1]/32	[35 21 7 1]/64
5	[35 56 28 8 1]/128	[126 84 36 9 1]/256
6	[126 210 120 45 10 1]/512	[462 330 165 55 11 1]/1024
7	[462 792 495 220 66 12 1]/2048	[1716 1287 715 286 78 13 1]/4096
8	[1716 3003 2002 1001 364 91 14 1]/8192	[6435 5005 3003 1365 455 105 15 1]/16384

Hamming

Minimum sidelobe

	Odd
2	[0.5383553946707251 0.4616446053292749]
3	[0.4243800934609435 0.4973406350967378 7.827927144231873e-2]
4	[0.3635819267707608 0.4891774371450171 0.1365995139786921 ... 1.064112210553003e-2]
5	[0.3232153788877343 0.4714921439576260 0.1755341299601972 ... 2.849699010614994e-2 1.261357088292677e-3]
6	[0.2935578950102797 0.4519357723474506 0.2014164714263962 ... 4.792610922105837e-2 5.02619642859393e-3 1.375555679558877e-4]
7	[0.2712203605850388 0.4334446123274422 0.2180041228929303 6.578534329560609e-2 1.076186730534183e-2 7.700127105808265e-4 1.368088305992921e-5]
8	[0.2533176817029088 0.4163269305810218 0.2288396213719708 ... 8.157508425925879e-2 1.773592450349622e-2 2.096702749032688e-3 1.067741302205525e-4 1.280702090361482e-6]

von Hann Window Frequency Response Figure 3

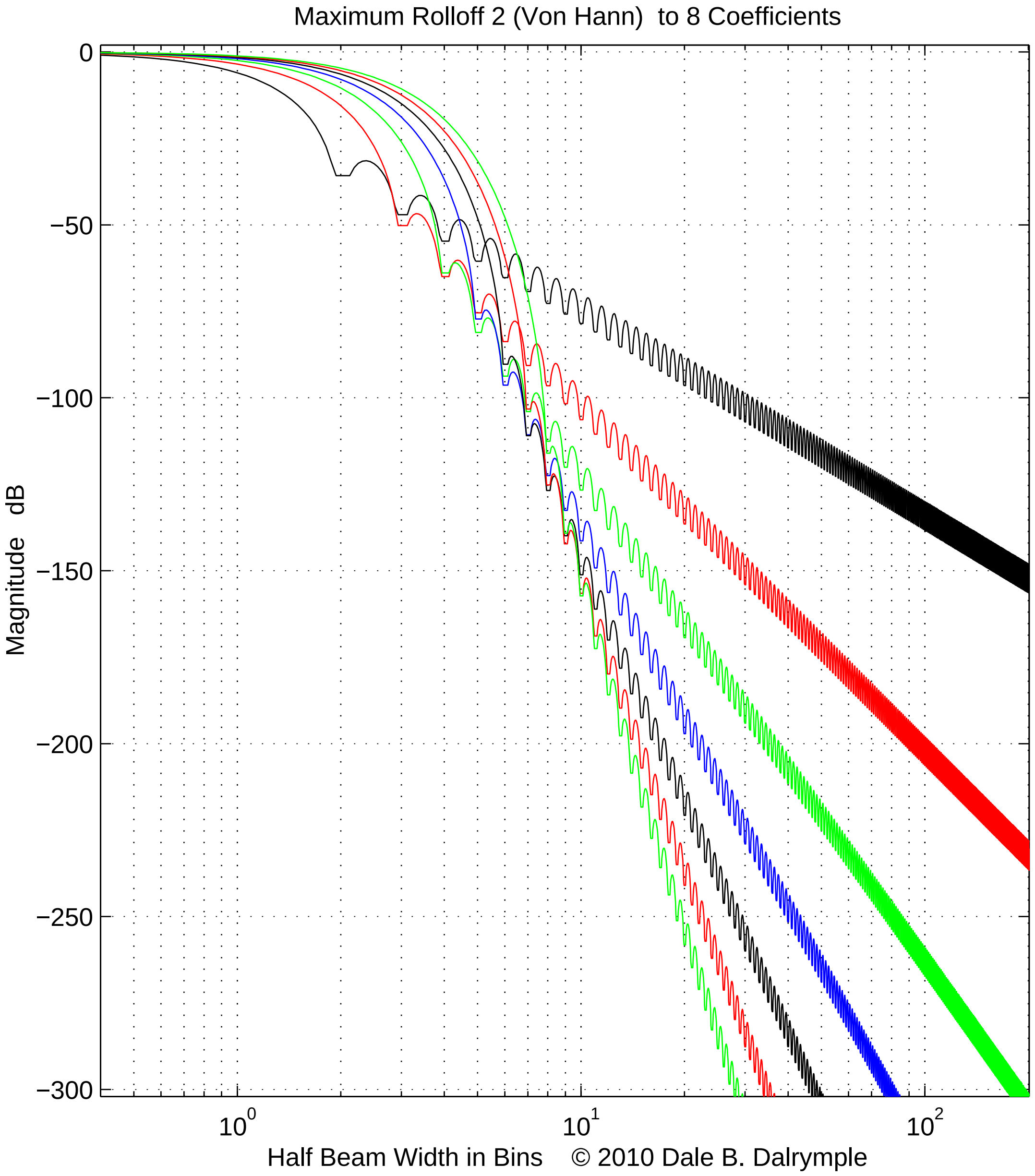


Maximum Sidelobe Rolloff Table

(2048 PtFFT) Param:	Coherent Gain	ENBW Bins	Scalloping Loss dB	WCPL dB	-3dB BW Bins	-6dB BW Bins	MSL Level dB	MSL BW Bins	SLRO dB/Oct	Bin1 Intercept dB
2,1	0.5	1.5	-1.424	3.185	1.453	2.02	-31.47	3.75	-18.1	-10
2,0	0.4244	1.735	-1.075	3.468	1.672	2.31	-39.3	4.78	-24.1	-5
3,1	0.375	1.944	-0.863	3.751	1.859	2.59	-46.74	5.8	-30.1	2.1
3,0	0.3395	2.135	-0.721	4.014	2.031	2.84	-53.93	6.8	-36.1	10.9
4,1	0.3125	2.31	-0.618	4.254	2.203	3.08	-60.95	7.81	-42.1	21.2
4,0	0.291	2.473	-0.541	4.474	2.359	3.3	-67.83	8.81	-48.2	32.7
5,1	0.2734	2.627	-0.482	4.675	2.5	3.5	-74.61	9.83	-54.2	45.5
5,0	0.2587	2.772	-0.434	4.861	2.625	3.69	-81.31	10.8	-60.2	59.1
6,1	0.2461	2.909	-0.394	5.032	2.766	3.88	-87.94	11.8	-66	
6,0	0.2352	3.041	-0.361	5.192	2.875	4.05	-94.52	12.8	-72	
7,1	0.2256	3.167	-0.334	5.341	3	4.22	-101.05	13.8	-78	
7,0	0.2171	3.289	-0.31	5.48	3.109	4.38	-107.54	14.8	-84	
8,1	0.2095	3.406	-0.289	5.612	3.219	4.53	-114	15.8	-90	
8,0	0.2026	3.519	-0.271	5.736	3.328	4.69	-120.43	16.9	-96	

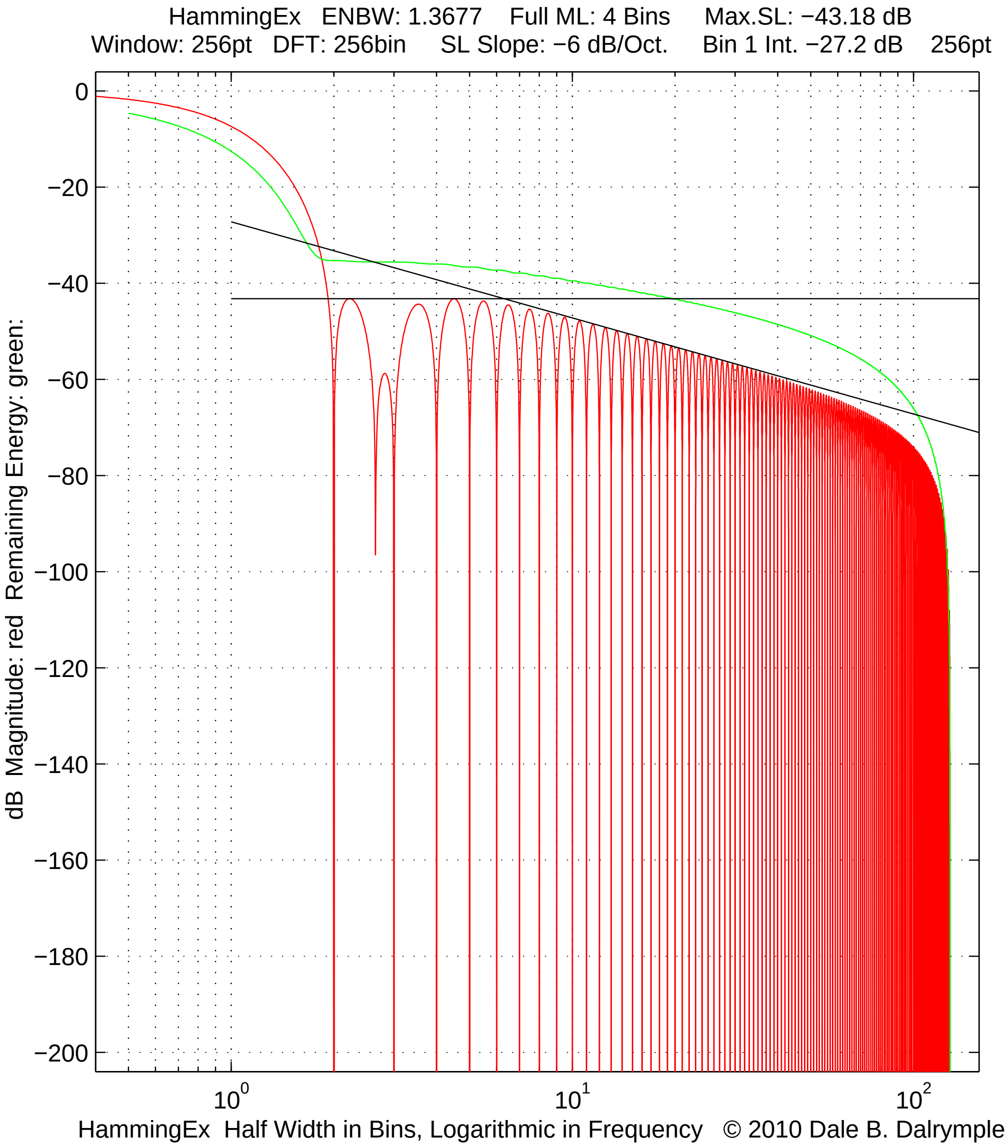
This table alternates odd and even cosine sum windows.

Maximum Sidelobe Rolloff Frequency Responses Figure 4



These are all **odd** cosine sum maximum sidelobe rolloff windows. (Median filter has been used to remove clutter of negative tails.)

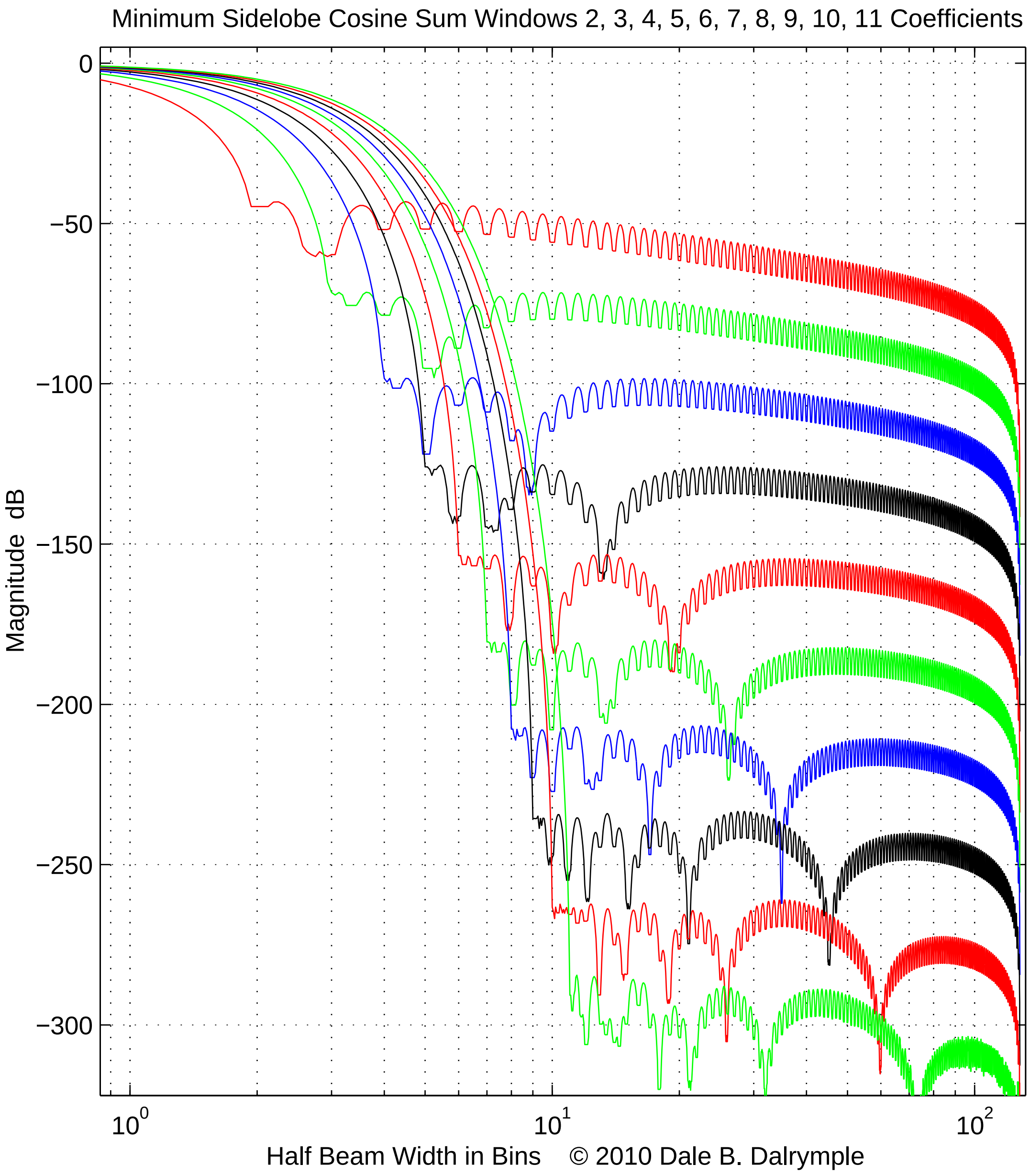
Hamming Window Frequency Response: Exact Hamming Figure 5



Minimim Sidelobe Table

(2048 Pt FFT) Param:	Coherent Gain	ENBW Bins	Scalloping Loss dB	WCPL dB	-3dB BW Bins	-6dB BW Bins	MSL Level dB	MSL BW Bins	SLRO dB/Oct	Bin1 Intercept dB
2	0.5384	1.368	-1.739	3.098	1.313	1.83	-43.19	3.86	-6	-27.1
3	0.4244	1.704	-1.135	3.449	1.625	2.28	-71.48	5.92	-6	-48.2
4	0.3636	1.976	-0.851	3.809	1.875	2.64	-98.17	7.94	-6	-70.2
5	0.3232	2.215	-0.68	4.134	2.109	2.95	-125.42	9.95	-6	-93.7
6	0.2936	2.434	-0.565	4.428	2.313	3.25	-153.56	12	-6	-118.7
7	0.2712	2.63	-0.485	4.685	2.484	3.5	-180.46	14	-6	-143.1
8	0.2533	2.813	-0.425	4.917	2.656	3.75	-207.5	16	-6	
9	0.2384	2.986	-0.378	5.129	2.828	3.98	-234.71	18	-6	
10	0.2257	3.152	-0.339	5.325	2.984	4.2	-262.84	20	-6	
11	0.2152	3.305	-0.309	5.501	3.125	4.41	-284.02	22	-6	

Minimum Sidelobe Frequency Responses Figure 6



These are all odd cosine sum windows.
(Median filter has been used to remove clutter of negative tails.)

How can we reduce sidelobes by adding additional components to the sinc function?

Blackman

Contribution: zero sidelobe centers, odd number of frequency domain terms
Solve a linear system of equations to normalize and place zeros at centers of highest sidelobes to reduce the height of the sidelobes.

R. B. Blackman and J. W. Tukey, *The Measurement of Power Spectra*, New York: Dover, 1958

Malocha and Bishop

Contribution: even and odd terms
Even numbers of terms provide frequency response intermediate between odd term responses. The even derived responses represent frequencies between original bin centers.

D. C. Malocha and C. D. Bishop, *"The classical truncated cosine series functions and applications to SAW filters," IEEE Trans. Ultrason., Ferroelect., Freq. Contr.*, vol. UFFC-34, pp 75-85, Jan. 1987

Kulkarni and Lahiri

Contribution: sidelobe peak zeros
The sinc function is the product of a sine wave and a hyperbola. The hyperbolic function moves the peaks away from the center of the sidelobes. Solve a linear system of equations to normalize and place zeros at peaks of highest sidelobes. (Only examined first sidelobe response.)

R. G. Kulkarni and S. K. Lahiri, *"Improved sidelobe performance of cosine series functions," IEEE Trans. Ultrason., Ferroelect., Freq. Contr.*, vol. 46, pp 464-466, Mar. 1999

Generalized Blackman Table

(2048 PtFFT) Param:	Coherent Gain	ENBW Bins	Scalloping Loss dB	WCPL dB	-3dB BW Bins	-6dB BW Bins	1st SL Level	MSL Level dB	MSL BW Bins	SLRO dB/Oct	Bin1 Intercept dB
2,1,B	0.5435	1.353	-1.778	3.091	1.297	1.81	**-46.01**	**-41.69**	3.81	**-6**	-26.1
2,1,K	0.5458	1.346	-1.796	3.087	1.297	1.8	**-47.51**	**-41.09**	3.78	**-6**	-25.7
3,1,B	0.4266	1.694	-1.15	3.438	1.609	2.27	-69.42	-68.24	5.89	**-6**	-46
3,1,K	0.4292	1.684	-1.164	3.426	1.609	2.25	-72.75	-67.47	5.86	**-6**	-45.4
4,1,B	0.363	1.978	-0.85	3.812	1.875	2.64	-91.34	-91.34	7.91	**-6**	-67.3
4,1,K	0.3651	1.967	-0.859	3.797	1.875	2.63	-96.01	-93.05	7.91	**-6**	-66.6
5,1,B	0.3213	2.227	-0.674	4.151	2.109	2.97	-112.61	-112.6	9.92	**-6**	-89.3
5,1,K	0.323	2.216	-0.681	4.136	2.109	2.95	-118.28	-118.22	9.94	**-6**	-88.5
6,1,B	0.2914	2.45	-0.558	4.451	2.328	3.27	**-133.5**	**-133.49**	11.9	**-6**	-111.7
6,1,K	0.2927	2.44	-0.563	4.436	2.313	3.25	**-140.02**	**-140.02**	11.9	**-6**	-110.9
2,0,B	0.463	1.573	-1.32	3.288	1.516	2.11	-54.99	-53.89	4.88	**-12**	-23.9
2,0,K	0.4651	1.566	-1.332	3.28	1.5	2.09	-56.87	-52.95	4.84	**-12**	-23.4
3,0,B	0.3849	1.872	-0.945	3.667	1.781	2.5	-78.28	-78.26	6.91	**-12**	-40.9
3,0,K	0.3871	1.862	-0.955	3.653	1.781	2.48	-81.92	-81.9	6.92	**-12**	-40.3
4,0,B	0.3364	2.131	-0.734	4.019	2.031	2.84	-100.12	-100.1	8.91	**-12**	-60
4,0,K	0.3382	2.12	-0.741	4.005	2.016	2.83	-105.04	-105.02	8.94	**-12**	-59.3
5,0,B	0.3025	2.363	-0.599	4.334	2.234	3.16	-121.34	-121.32	10.9	**-12**	-80.2
5,0,K	0.304	2.352	-0.605	4.319	2.234	3.14	-127.26	-127.23	10.9	**-12**	-79.4
6,0,B	0.2772	2.574	-0.506	4.613	2.438	3.44	-142.19	-142.17	12.9	**-12**	-101.2
6,0,K	0.2783	2.564	-0.511	4.599	2.422	3.42	-148.94	-148.91	12.9	**-12**	-100.3

This table includes odd (at top) and even (at bottom) cosine sum windows.
Window parameter B indicates Blackman (lobe center zero), K indicates Kulkarni (sinc lobe peak zero).

2, 3 and 4 Term Cosine Sum Table

(2048 PtFFT) Param:	Coherent Gain	ENBW Bins	Scalloping Loss dB	WCPL dB	-3dB BW Bins	-6dB BW Bins	MSL Level dB	MSL BW Bins	SLRO dB/Oct	Bin1 Intercept dB
Rectangular	1.0	1.0	-3.922	3.922	0.890	1.22	-13.26	3.28	-6	-10.2
von Hann	0.5	1.5	-1.424	3.185	1.453	2.02	-31.47	3.75	-18.1	-10
HammingEx	0.5384	1.368	-1.739	3.098	1.313	1.83	-43.19	3.86	-6	-27.1
HammingRnd2	0.54	1.363	-1.751	3.096	1.313	1.83	-42.68	3.84	-6	-26.8
CS3	0.4496	1.611	-1.267	3.337	1.547	2.16	-62.05	5.39	-6	-41.1
CS3C1D	**0.409**	**1.772**	**-1.045**	**3.53**	**1.688**	**2.36**	**-64.19**	**5.89**	**-18.1**	**-19.5**
CS3D3D	0.375	1.944	-0.863	3.751	1.859	2.59	-46.74	5.8	-30.1	2.1
CS3min	0.4244	1.704	-1.135	3.449	1.625	2.28	-71.48	5.92	-6	-48.2
CS3minharris	0.4232	1.709	-1.129	3.455	1.625	2.28	-70.83	5.91	-6	-48.9
CS4	0.4022	1.794	-1.027	3.565	1.703	2.39	-72.43	6.39	-6	-55.3
CS4C1D	0.3558	2.021	-0.812	3.868	1.922	2.7	-93.32	7.94	-18.1	-33.4
CS4C3D	0.3389	2.125	-0.732	4.006	2.031	2.83	-82.6	7.91	-30.1	-4.5
CS4C5D	0.3125	2.31	-0.618	4.254	2.203	3.08	-60.95	7.81	-42.1	21.2
CS4min	0.3636	1.976	-0.851	3.809	1.875	2.64	**-98.17**	7.94	-6	-70.2
CS4minharris	0.3588	2.004	-0.826	3.845	1.906	2.67	**-92.01**	7.89	-6	-85.8
Blackman3Ex	0.4266	1.694	-1.15	3.438	1.609	2.27	**-68.24**	5.89	-6	-46
BlackmanNVSP	**0.42**	**1.727**	**-1.099**	**3.471**	**1.656**	**2.31**	**-58.11**	**5.66**	**-18.1**	**-17.3**
Blackman3Rnd3	0.426	1.697	-1.145	3.441	1.625	2.27	-70.04	5.89	-6	-47.2

Notes on bold areas: **CS3C1D** this is an improved replacement if someone told you 0.42 0.5 0.08 is called "Blackman" and you like it. **-98.17** dB is the best sidelobe rejection for a 4 term window, **-92.01** dB is the best that 4-term Blackman-harris can do. 4-term Blackman-harris is not a minimum sidelobe window. **-68.24** dB is the correct sidelobe rejection for 2 digit rounded 3-term Blackman which Blackman and Tukey called "Blackman's not very serious proposal". The harris paper incorrectly gives this as -51 dB which may have unduly popularized the rounded version. Use CS3C1D from Nuttall instead of the rounded version for -18 dB/octave sidelobe rolloff and better minimum sidelobe rejection.

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Other Optimizations

Any time you hear "optimum" or "optimized" be sure you have heard "with respect to" what.

What: Mini-max stopband error. What window gives the smallest maximum error outside the passband?
Dolph-Chebychev

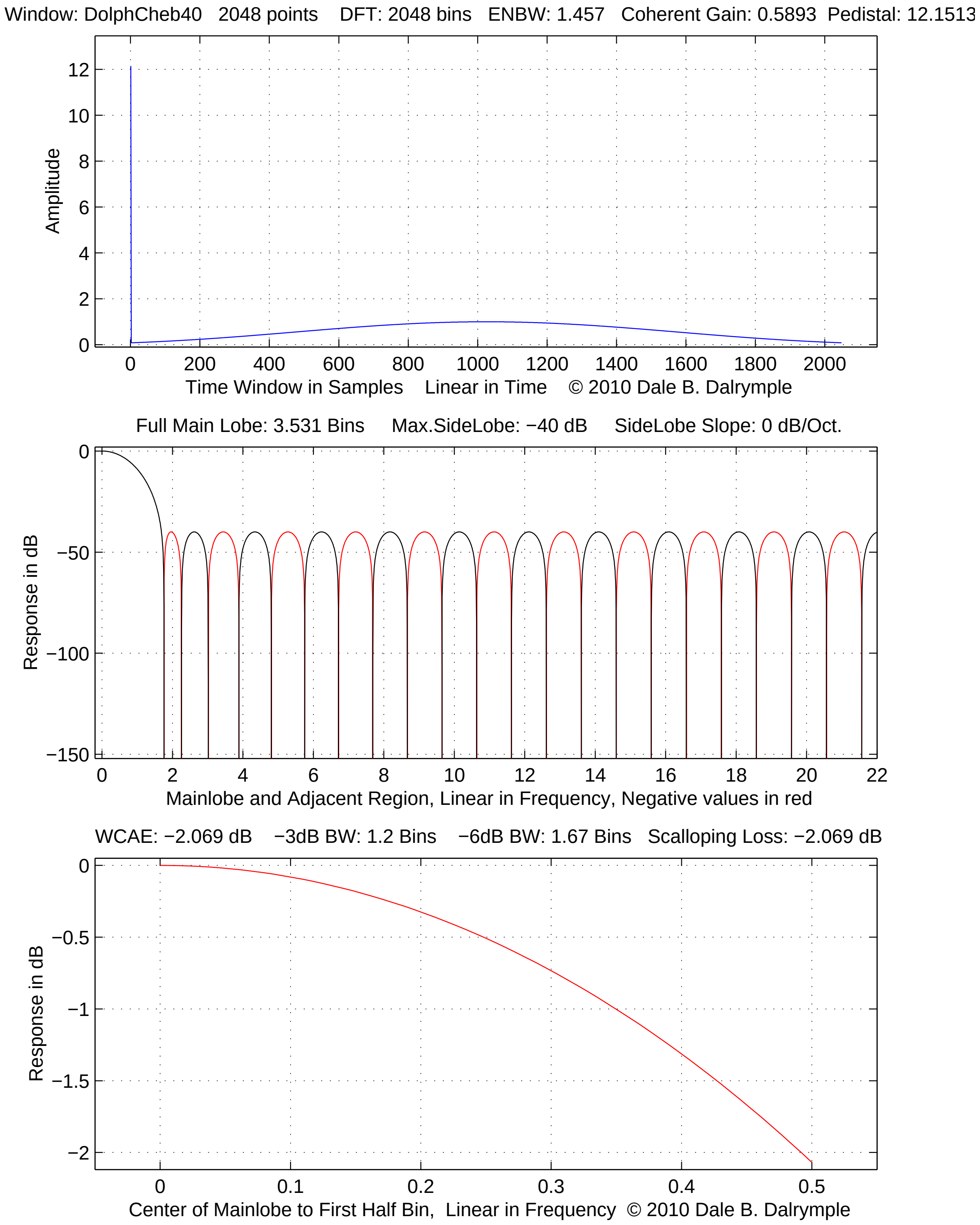
Approximation to Dolph-Chebychev:
Taylor (two parameter)

What: Minimum stopband energy. What window gives the smallest total energy outside the passband?
Prolate Spheroidal Functions implemented as **Discrete Prolate Spheroidal Sequences (DPSS)**

Approximation to Prolate spheroidal functions:
Io(a): Modified Bessel Function of the First Kind of Order Zero
also know as "Taylor 1 parameter" and "Kaiser-Bessel"

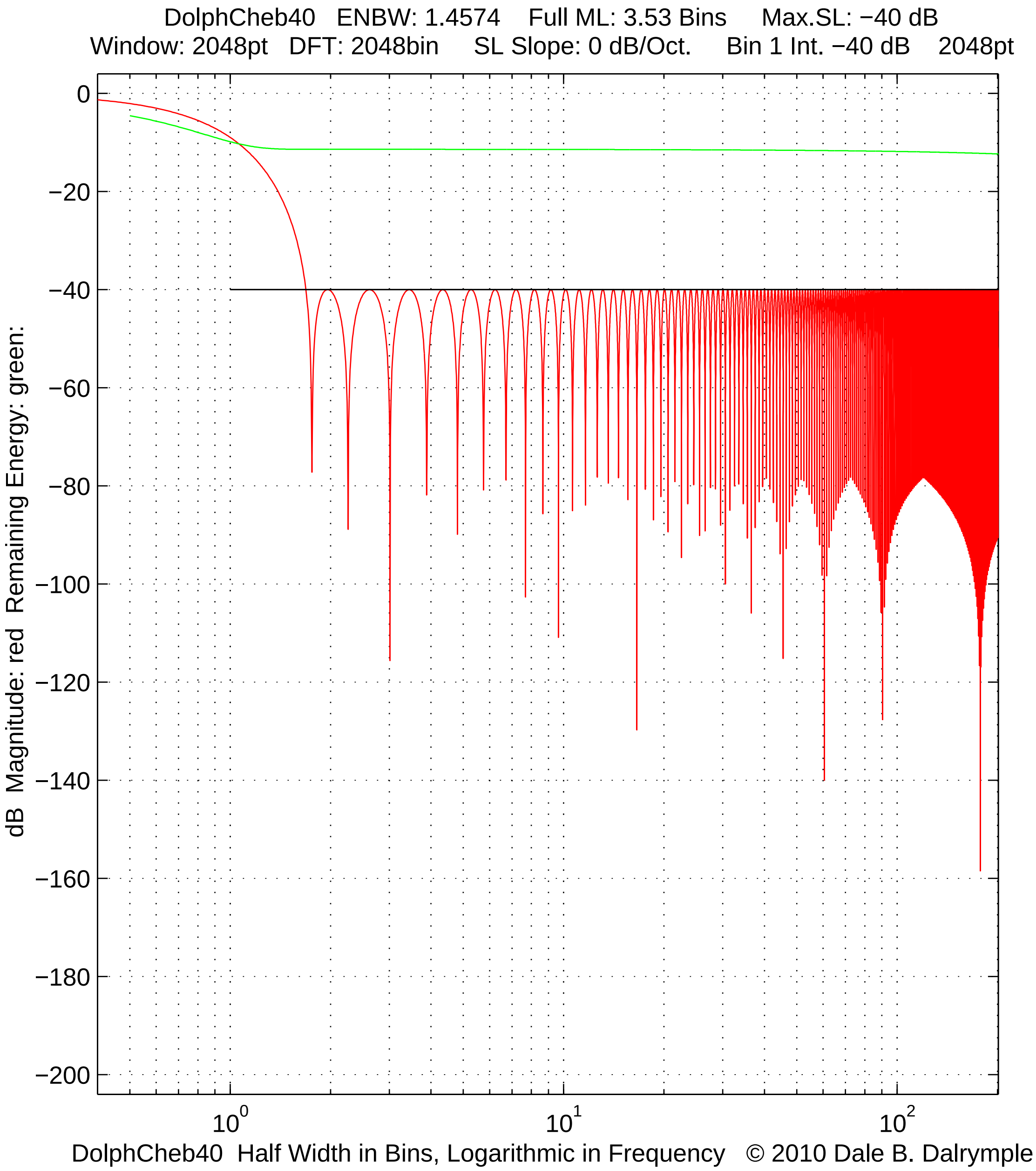
An improvement to "Kaiser"-Bessel!
"Modified Kaiser Bessel"

Dolph-Chebychev 40dB Plots Figure 7

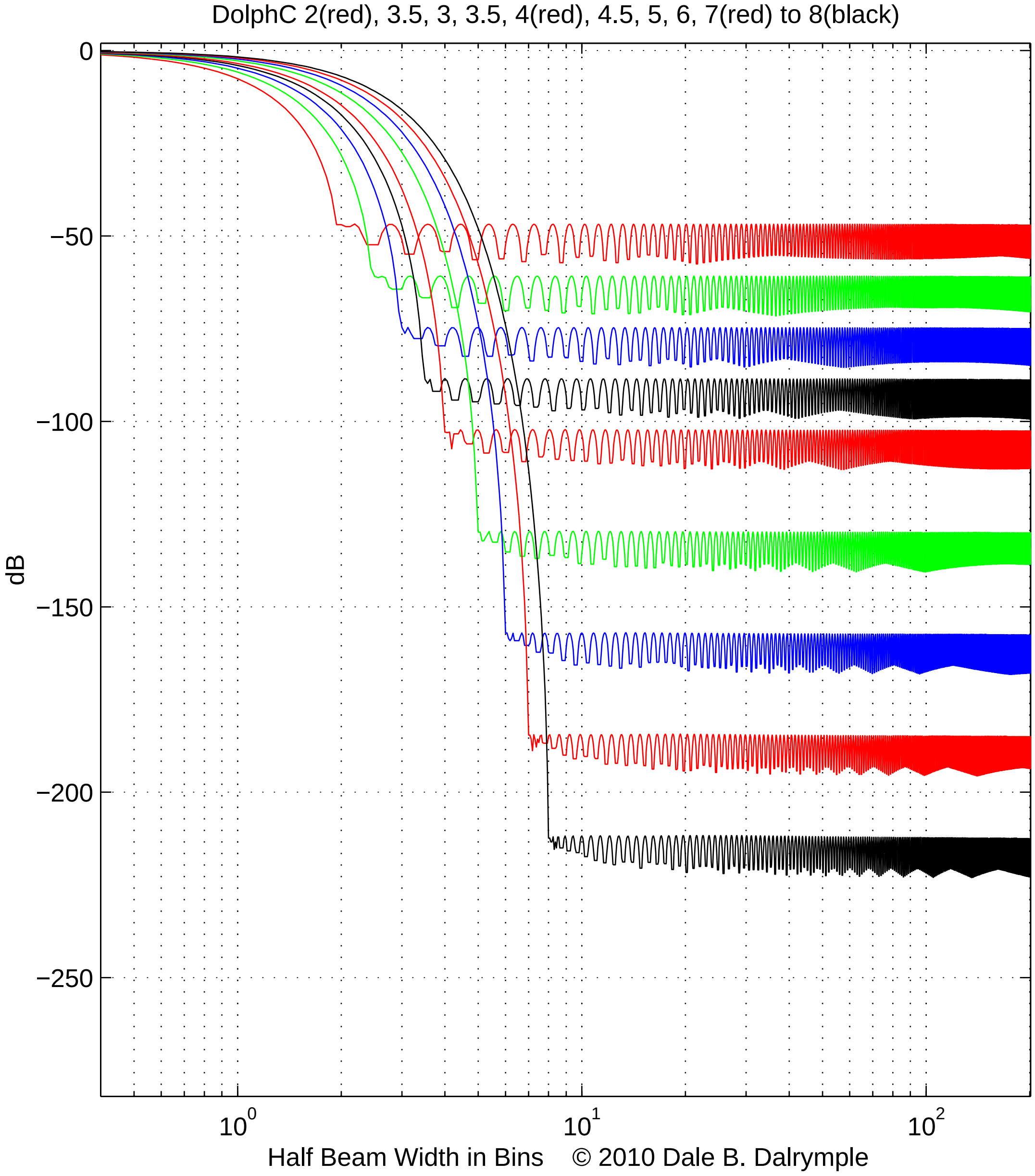


High peak at end of time window.
Zero sidelobe rolloff.

Dolph-Chebychev 40dB Frequency Response Figure 8



Dolph-Chebychev Frequency Responses Figure 9



Dolph-Cheb Table

(2048 PtFFT) Param:	Coherent Gain	ENBW Bins	Scalloping Loss dB	WCPL dB	-3dB BW Bins	-6dB BW Bins	MSL Level dB	MSL BW Bins	SLRO dB/Oct	Bin1 Intercept dB	Window Pedestal
20	0.8502	**21.4**	-3.836	17.1	0.906	1.23	-20	1.91	0	-20	**174.5124**
30	0.6838	**3.151**	-2.682	7.667	1.063	1.47	-30	2.64	0	-30	**44.4716**
40	0.5893	1.457	-2.069	3.705	1.203	1.67	-40	3.38	0	-40	**12.1513**
50	0.5258	1.411	-1.687	3.181	1.328	1.86	-50	4.11	0	-50	**3.4402**
60	0.4794	1.518	-1.425	3.239	1.453	2.03	-60	4.84	0	-60	0.99568
70	0.4434	1.633	-1.234	3.364	1.563	2.19	-70	5.58	0	-70	0.29258
80	0.4145	1.741	-1.089	3.498	1.656	2.33	-80	6.31	0	-80	0.086947
90	0.3906	1.844	-0.974	3.631	1.75	2.45	-90	7.05	0	-90	0.026064
100	0.3704	1.94	-0.882	3.761	1.844	2.59	**-100**	**7.78**	0	-100	0.0078675
110	0.3531	2.033	-0.805	3.886	1.938	2.72	-110	8.52	0	-110	0.0023885
120	0.338	2.121	-0.741	4.006	2.016	2.83	-120	9.25	0	-120	0.00072868
130	0.3247	2.206	-0.686	4.121	2.094	2.94	-130	9.98	0	-130	0.00022323
140	0.3128	2.287	-0.639	4.232	2.172	3.05	-140	10.7	0	-140	6.8636e-005
150	0.3021	2.366	-0.598	4.338	2.25	3.16	-150	11.4	0	-150	2.1172e-005
160	0.2925	2.442	-0.562	4.439	2.313	3.25	-160	12.2	0	-160	6.5496e-006
170	0.2838	2.516	-0.53	4.537	2.391	3.36	-170	12.9	0	-170	2.0315e-006
180	0.2757	2.588	-0.501	4.63	2.453	3.45	-180	13.6	0	-180	6.316e-007
190	0.2684	2.658	-0.475	4.721	2.516	3.55	-189.97	14.4	0	-190	1.9682e-007
200	0.2615	2.726	-0.452	4.807	2.578	3.63	-199.93	15.1	0	-200	6.1455e-008

Taylor (2 Parameter) Window Maximum nbar for Monotonic Time Domain Window
Tabulated for Sidelobe Rejection vs Transform Size

Taylor Window with nbar in **bold** should be used to replace Dolph-Cheb.

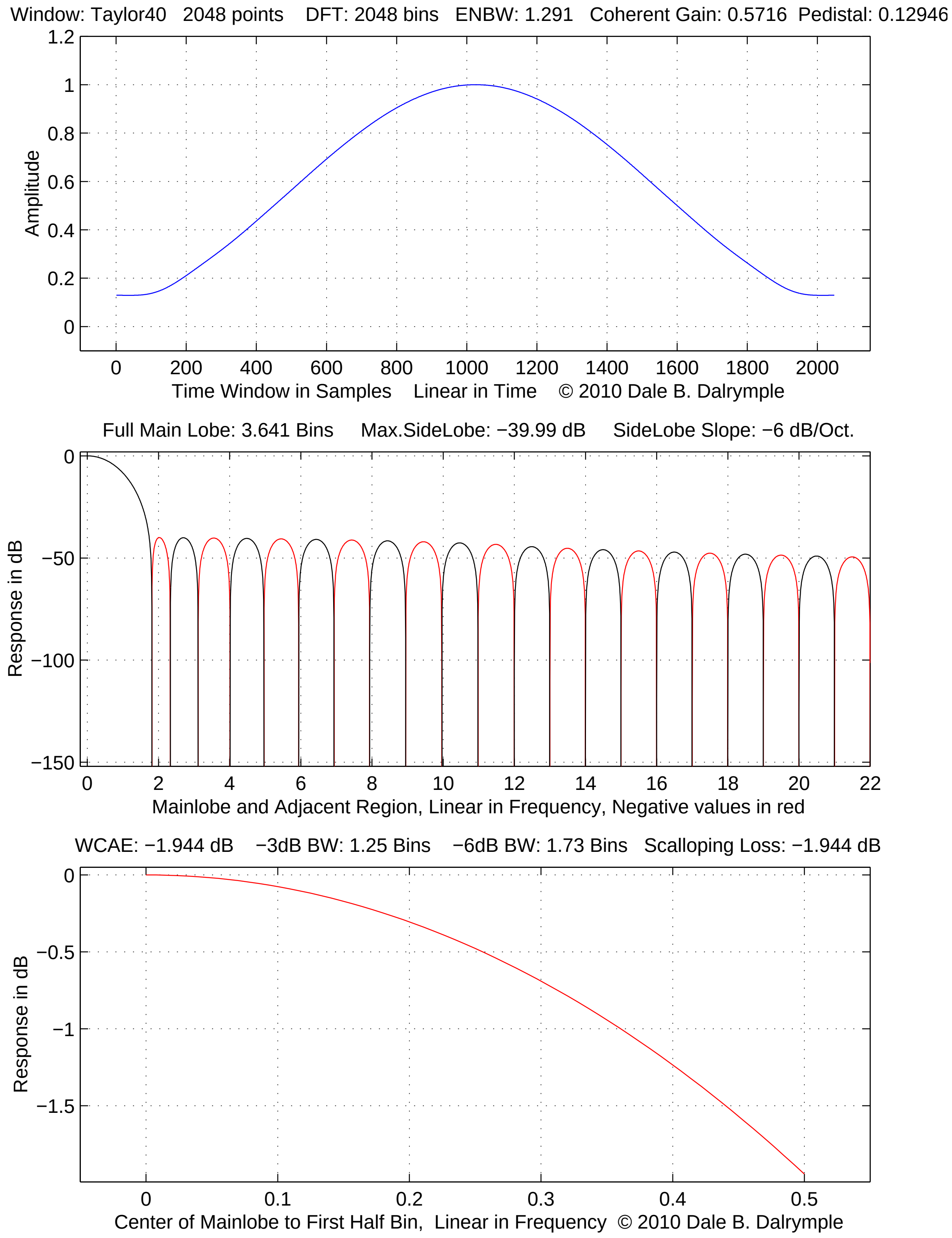
XformSize SLRR	32	64	128	256	512	1024	2048
20	**4**	**4**	**4**	**4**	**4**	**4**	**4**
25	**6**	**6**	**6**	**6**	**6**	**6**	**6**
30	**8**	**8**	**8**	**8**	**8**	**8**	**8**
35	**10**	**10**	**10**	**10**	**10**	**10**	**10**
40	**14**	**12**	**12**	**12**	**12**	**12**	**12**
45	16	**15**	**15**	**15**	**15**	**15**	**15**
50	16	**19**	**18**	**18**	**18**	**18**	**18**
55	16	**22**	**21**	**21**	**21**	**21**	**21**
60	16	**28**	**25**	**24**	**24**	**24**	**24**
70	16	32	**33**	**32**	**32**	**32**	**32**
80	16	32	**44**	**41**	**41**	**41**	**41**
100	16	32	64	**64**	**62**	**61**	**61**
110	16	32	64	**78**	**74**	**73**	**73**
120	16	32	64	**95**	**88**	**86**	**86**
130	16	32	64	**119**	**103**	**101**	**100**
140	16	32	64	128	**120**	**116**	**116**
150	16	32	64	128	**138**	**134**	**132**
160	16	32	64	128	**160**	**152**	**150**
170	16	32	64	128	**184**	**170**	**168**
180	16	32	64	128	**214**	**192**	**188**
190	16	32	64	128	**254**	**214**	**210**
200	16	32	64	128	256	**238**	**232**

Taylor Window Table

(2048 PtFFT) Param:	Coherent Gain	ENBW Bins	Scalloping Loss dB	WCPL dB	-3dB BW Bins	-6dB BW Bins	MSL Level dB	MSL BW Bins	SLRO dB/Oct	Bin1 Intercept dB	Window Pedestal
20, 4	0.7785	**1.041**	-3.154	3.329	0.984	1.34	-20.42	2.11	-6	-12.5	**0.59553**
30, 8	0.6526	**1.156**	-2.434	3.063	1.109	1.55	-30.1	2.78	-6	-16.9	**0.30194**
40,12	0.5716	1.291	-1.944	3.054	1.25	1.73	-39.99	3.48	-6	-23.1	**0.12946**
50,18	0.5153	1.419	-1.619	3.138	1.359	1.89	-49.94	4.2	-6	-29.7	**0.054719**
60,24	0.4723	1.539	-1.383	3.255	1.469	2.06	-59.89	4.92	-6	-37	0.021508
70,32	0.4385	1.651	-1.207	3.384	1.578	2.2	-69.82	5.64	-6	-44.5	0.0083792
80,41	0.411	1.756	-1.07	3.516	1.672	2.34	-79.74	6.38	-6	-52.4	0.0031762
90,51	0.388	1.856	-0.961	3.647	1.766	2.48	-89.66	7.09	-6	-60.5	0.0011793
100,61	0.3684	1.951	-0.872	3.775	1.859	2.61	**-99.54**	**7.81**	-6	-68.9	0.00042724
110,73	0.3515	2.042	-0.798	3.898	1.938	2.72	-109.48	8.55	-6	-77.3	0.00015423
120,85	0.3367	2.129	-0.735	4.017	2.016	2.84	-119.36	9.28	-6	-85.9	5.4738e-005
130,100	0.3236	2.213	-0.682	4.131	2.094	2.95	-129.28	10	-6	-94.6	1.9471e-005
140,116	0.3119	2.293	-0.635	4.24	2.172	3.06	-139.16	10.7	-6	-103.3	6.8604e-006
150,132	0.3014	2.371	-0.595	4.345	2.25	3.16	-149.01	11.5	-6	-112.1	2.3889e-006
160,150	0.2919	2.447	-0.559	4.446	2.313	3.27	-158.92	12.2	-6	-121	8.2972e-007
170,168	0.2833	2.52	-0.528	4.543	2.391	3.36	-168.79	12.9	-6	-129.9	2.8561e-007
180,192	0.2753	2.592	-0.5	4.635	2.453	3.45	-178.63	13.7	-6	-138.4	9.9175e-008
190,210	0.268	2.661	-0.474	4.725	2.516	3.55	-188.48	14.4	-6	-147.8	3.3636e-008
200,232	0.2612	2.729	-0.451	4.811	2.578	3.64	-198.32	15.1	-6	-157.4	1.1452e-008

Villeneuve,A. T., *“Taylor Patterns for Discrete Arrays”*
Antennas and Propagation, Transactions on; vol. AP-32, no. 10, October 1984

Taylor 40dB Plots Figure 10

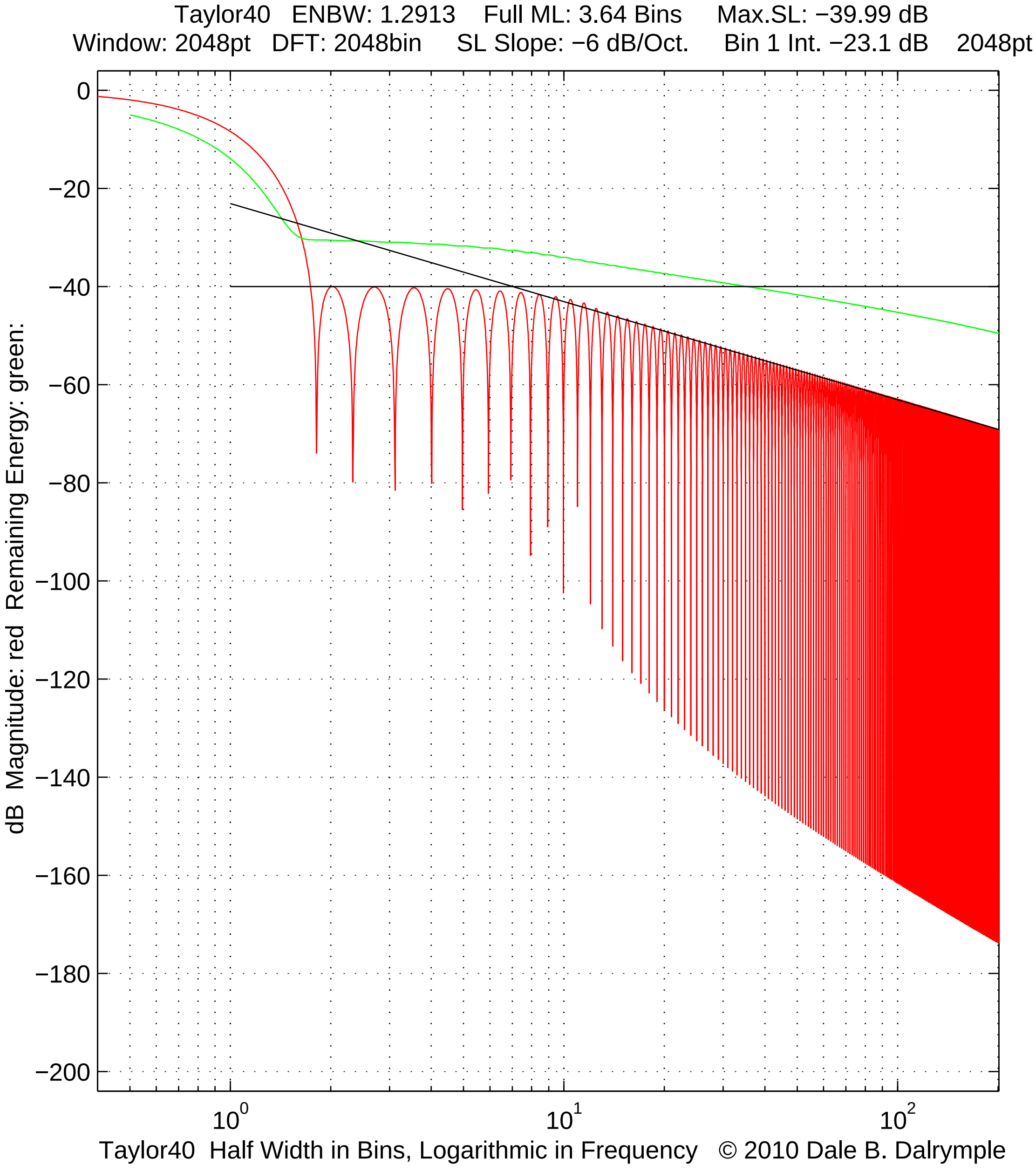


Window: Taylor40 2048 points DFT: 2048 bins ENBW: 1.291 Coherent Gain: 0.5716 Pedistal: 0.12946

Full Main Lobe: 3.641 Bins Max.SideLobe: −39.99 dB SideLobe Slope: −6 dB/Oct.

WCAE: −1.944 dB −3dB BW: 1.25 Bins −6dB BW: 1.73 Bins Scalloping Loss: −1.944 dB

Taylor 40dB Frequency Response Figure 11



DPSS Table

(2048 PtFFT) Param:	Coherent Gain	ENBW Bins	Scalloping Loss dB	WCPL dB	-3dB BW Bins	-6dB BW Bins	MSL Level dB	MSL BW Bins	SLRO dB/Oct	Bin1 Intercept dB
20	0.7537	1.073	-2.857	3.163	1.031	1.42	-20.14	2.22	-6	-16.9
30	0.5852	1.277	-1.972	3.034	1.234	1.7	-32.19	3.19	-6	-28.3
40	0.5215	1.411	-1.627	3.122	1.359	1.89	-40.93	3.89	-6	-36.5
50	0.4747	1.537	-1.38	3.248	1.469	2.06	-50.14	4.61	-6	-45.1
60	0.4386	1.655	-1.197	3.385	1.578	2.22	-59.63	5.34	-6	-54.1
70	0.4096	1.765	-1.057	3.525	1.688	2.36	-69.26	6.08	-6	-63.2
80	0.3857	1.869	-0.946	3.663	1.781	2.5	-78.99	6.83	-6	-72.5
90	0.3656	1.968	-0.856	3.796	1.875	2.63	-88.81	7.58	-6	-81.9
100	0.3483	2.062	-0.781	3.925	1.953	2.75	-98.74	8.33	-6	-91.3
120	0.32	2.239	-0.665	4.165	2.125	2.98	-118.76	9.84	-6	-110.5
140	0.2976	2.402	-0.579	4.386	2.281	3.2	-138.81	11.4	-6	-129.8
160	0.2794	2.556	-0.513	4.588	2.422	3.41	-159.04	12.9	-6	-149.3
180	0.2641	2.7	-0.46	4.775	2.563	3.59	-179.35	14.4	-6	-168.9
200	0.2511	2.838	-0.418	4.947	2.688	3.78	**-199.6**	**15.9**	-6	-188.6

Kaiser Window Table

(2048 PtFFT) Param:	Coherent Gain	ENBW Bins	Scalloping Loss dB	WCPL dB	-3dB BW Bins	-6dB BW Bins	MSL Level dB	MSL BW Bins	SLRO dB/Oct	Bin1 Intercept dB
1	0.6711	1.152	-2.428	3.041	1.109	1.53	-24.57	2.58	-6	-21.5
1.5	0.56	1.327	-1.83	3.059	1.281	1.78	-34.71	3.42	-6	-31.6
2	0.4892	1.496	-1.453	3.203	1.438	2	-45.85	4.33	-6	-42.8
2.5	0.4396	1.652	-1.201	3.381	1.578	2.2	-57.56	5.27	-6	-54.5
3	0.4025	1.795	-1.023	3.564	1.719	2.39	-69.62	6.22	-6	-66.5
3.5	0.3735	1.928	-0.89	3.742	1.844	2.58	-81.92	7.19	-6	-78.8
4	0.3499	2.053	-0.788	3.912	1.953	2.73	**-94.41**	**8.17**	**-6**	-91.3
4.5	0.3303	2.171	-0.706	4.073	2.063	2.89	-107.03	9.16	-6	-103.9
5	0.3136	2.283	-0.64	4.225	2.172	3.05	-119.76	10.1	-6	-116.7
5.5	0.2993	2.39	-0.585	4.369	2.266	3.19	-132.58	11.1	-6	-129.5
6	0.2867	2.492	-0.539	4.505	2.359	3.33	-145.47	12.1	-6	-142.3
6.5	0.2756	2.59	-0.5	4.633	2.453	3.45	-158.41	13.1	-6	-155.3
7	0.2657	2.685	-0.466	4.755	2.547	3.58	-171.42	14.1	-6	-168.3
7.5	0.2568	2.776	-0.436	4.871	2.625	3.7	-184.46	15.1	-6	-181.3
8	0.2487	2.865	-0.41	4.981	2.703	3.81	**-197.54**	**16.1**	**-6**	-194.3
8.5	0.2414	2.951	-0.387	5.086	2.797	3.94	-210.66	17.1	-6	NaN
9	0.2346	3.034	-0.366	5.186	2.875	4.05	-223.82	18.1	-6	NaN

KaiserMod Table

(2048 PtFFT) Param:	Coherent Gain	ENBW Bins	Scalloping Loss dB	WCPL dB	-3dB BW Bins	-6dB BW Bins	MSL Level dB	MSL BW Bins	SLRO dB/Oct	Bin1 Intercept dB
2	0.4833	1.52	-1.401	3.221	1.469	2.03	-41.61	4.23	-12	-26.1
2.5	0.4381	1.66	-1.187	3.388	1.594	2.22	-55.82	5.19	-12	-34.1
3	0.4022	1.798	-1.019	3.566	1.719	2.41	-71.87	6.27	-12	-43.1
3.5	0.3734	1.929	-0.889	3.743	1.844	2.58	-84.28	7.25	-12	-52.7
4	0.3499	2.053	-0.787	3.912	1.953	2.73	**-92.5**	**8.16**	**-12**	-62.9
4.5	0.3303	2.171	-0.706	4.073	2.063	2.89	-105.16	9.13	-12	-73.5
5	0.3136	2.283	-0.64	4.225	2.172	3.05	-121.19	10.2	-12	-84.4
5.5	0.2993	2.39	-0.585	4.369	2.266	3.19	-134.68	11.2	-12	-95.5
6	0.2867	2.492	-0.539	4.505	2.359	3.33	-144.47	12.1	-12	-106.9
7	0.2657	2.685	-0.466	4.755	2.547	3.58	-172.23	14.1	-12	-130.2
8	0.2487	2.865	-0.41	4.981	2.703	3.81	**-196.96**	**16.1**	**-12**	-154.1
9	0.2346	3.034	-0.366	5.186	2.875	4.05	-224.31	18.1	NaN	NaN

How to “modify” Kaiser-Bessel? “Subtract 1.0 from numerator and denominator.”

Other Windows

Gaussian

The infinite continuous Gaussian window is the only window to achieve the minimum time-frequency uncertainty product. While this is not achieved for truncated Gaussians, it seems to be a frequently referenced attraction. The Gaussian also has interesting properties for frequency estimation of tones. The parabolic interpolator is very accurate for the Gaussian log power spectrum. There are also other techniques:

McEachern, Robert H , "*Ratio detection precisely characterizes signals' amplitude and frequency*", EDN, March 3, 1994
Available (20100404) at:
http://www.edn.com/archives/1994/030394/05df1.htm

Modified Bessel Functions of the First Kind I(order,alpha)

Order = 0	Kaiser, Taylor (1953)
Order = 0,1	Prabu
Order = n, alpha > 0 n = -2, -3/2, -1, -1/2, 0	Reddy
Order real, alpha real 1,2,3 dimensions	Nuttall

Kaiser, J. F. and R.W. Schafer, *"On the Use of the lo-Sinh Window for Spectrum Analysis",* IEEE Trans.Acoust., Speech, Signal Proc., ASSP-28, pplOS,1980.

Prabhu, K.M.M.; Bagan, K.B.; *" Variable parameter window families for digital spectral analysis"*
Acoustics, Speech and Signal Processing, IEEE Tran. on
Volume: 37 Issue:6, June 1989, page(s): 946 – 949

Reddy, A.R.; *"Design of SAW bandpass filters using new window functions"* Ultrasonics, Ferroelectrics and Frequency Control, IEEE Tran. on, Volume: 35 , Issue: 1 ,Publication Year: 1988 , Page(s): 50 - 56

Nuttall, A.; "*A two-parameter class of Bessel weightings for spectral analysis or array processing--The ideal weighting-window pairs* "
This paper appears in: Acoustics, Speech and Signal Processing, IEEE Transactions on, Volume: 31 Issue:5, page(s): 1309 - 1312

Gaussian Table

(2048 PtFFT) Param:	Coherent Gain	ENBW Bins	Scalloping Loss dB	WCPL dB	-3dB BW Bins	-6dB BW Bins	MSL Level dB	MSL BW Bins	SLRO dB/Oct	Bin1 Intercept dB
2	0.5981	1.233	-2.128	3.037	1.188	1.66	-31.89	3.11	-6	-23.1
2.5	0.4951	1.446	-1.58	3.181	1.375	1.92	-43.25	5.91	-6	-31.2
3	0.4166	1.702	-1.163	3.472	1.609	2.27	-56.07	6.69	-6	-41.7
3.5	0.3579	1.977	-0.87	3.829	1.859	2.63	-71	9.92	-6	-54.5
4	0.3133	2.257	-0.669	4.205	2.125	3	-87.61	11.1	-6	-69.6
4.5	0.2785	2.539	-0.529	4.576	2.391	3.38	-107.42	14.4	-6	-87
5	0.2507	2.821	-0.429	4.933	2.656	3.75	-128.64	18	-6	-106.7
5.5	0.2279	3.103	-0.354	5.272	2.922	4.13	-149.15	20.6	-6	-128.7
6	0.2089	3.385	-0.298	5.593	3.188	4.5	-175.88	24	-6	-152.9
6.5	0.1928	3.667	-0.254	5.897	3.453	4.88	-203.56	28.1	-6	NaN
7	0.179	3.949	-0.219	6.184	3.719	5.25	-235.35	33.9	-6	NaN

Extended Bessel Function Io(alpha ,order) Table

(2048 PtFFT) Param:	Coherent Gain	ENBW Bins	Scalloping Loss dB	WCPL dB	-3dB BW Bins	-6dB BW Bins	MSL Level dB	MSL BW Bins	SLRO dB/Oct	Bin1 Intercept dB	
1,-2	0.8031	1.042	-3.104	3.282	1	1.36	-18.2	2.05	-6	-14.8	
2,-2	0.5731	1.29	-1.939	3.046	1.25	1.73	-36.67	3.38	-6	-27.3	
3,-2	0.4498	1.607	-1.274	3.335	1.531	2.14	-59.87	5.8	-6	-44.6	
4,-2	0.3801	1.89	-0.93	3.695	1.797	2.52	-84.66	7.48	-6	-64.7	
1,-1	0.7507	1.074	-2.851	3.16	1.031	1.42	-20.33	2.22	-6	-16.8	
2,-1	0.5317	1.381	-1.7	3.102	1.328	1.84	-42.93	3.86	-6	-32.8	
3,-1	0.4257	1.698	-1.144	3.442	1.625	2.27	-69.82	6	-6	-53.1	
4,-1	0.3647	1.97	-0.856	3.8	1.875	2.63	-97.14	7.92	-6	-75.4	
1, 0	0.6711	1.152	-2.428	3.041	1.109	1.53	-24.57	2.58	-6	-21.5	"Kaiser"
2, 0	0.4892	1.496	-1.453	3.203	1.438	2	-45.85	4.33	-6	-42.8	"Kaiser"
3, 0	0.4025	1.795	-1.023	3.564	1.719	2.39	-69.62	6.22	-6	-66.5	"Kaiser"
4, 0	0.3499	2.053	-0.788	3.912	1.953	2.73	-94.41	8.17	-6	-91.3	"Kaiser"
1, 1	0.561	1.348	-1.759	3.057	1.313	1.81	-28.92	3.25	-12	-17.8	
2, 1	0.4488	1.629	-1.226	3.346	1.563	2.17	-45.96	4.75	-12	-34.8	
3, 1	0.3806	1.898	-0.915	3.698	1.813	2.53	-66.73	6.53	-12	-55.6	
4, 1	0.3357	2.14	-0.725	4.029	2.031	2.86	-89.28	8.41	-12	-78.1	
1, 2	0.485	1.537	-1.359	3.227	1.484	2.06	-33.43	3.94	-18.1	-12	
2, 2	0.4134	1.765	-1.047	3.514	1.688	2.36	-47.45	5.25	-18.1	-26	
3, 2	0.3603	2.004	-0.821	3.84	1.906	2.67	-65.72	6.89	-18.1	-44.3	
4, 2	0.3222	2.229	-0.669	4.15	2.125	2.97	-86.27	8.69	-18.1	-64.9	
1, 3	0.4307	1.714	-1.1	3.439	1.641	2.3	-37.89	4.64	-24.1	-3.9	
2, 3	0.383	1.9	-0.906	3.693	1.813	2.53	-49.71	5.8	-24.1	-15.7	
3, 3	0.3416	2.112	-0.74	3.987	2.016	2.81	-65.9	7.33	-24.1	-31.9	
4, 3	0.3095	2.32	-0.617	4.272	2.203	3.09	-84.67	9.03	-24.1	-50.7	

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