One-Bit Delta Sigma D/A Conversion
Part I: Theory

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Contents

1 What Is A D/A Converter?

- Rick Lyons [1] derives A/D SNR as a function of word length N and loading factor LF:

$$\text{SNR} = 6.02N + 4.77 + 20\log_{10}(LF),$$

- LF is the "loading factor," a value representing the normalized RMS value of the input signal. For a sine wave, $LF = 0.707$. Here we ignore the constant factor of 1.77 dB and we round the N coefficient to 6 to simplify.

- This can be generalized to express the SNR of any N-bit amplitude-quantized transfer function and thus applies to D/A conversion as well.

For a generic D/A converter in which bandwidth, output bit-width, and other parameters may not be clearly defined, this motivates the following

Definition 1 *An* ***N-bit D/A converter*** *converts a stream of discrete-time, linear, PCM samples of* N *bits at sample rate* F_s *to a continuous-time analog voltage with a signal-to-quantization-noise power ratio of* $6N$ *dB in a bandwidth of* $F_s/2$ *Hz.*

This gives a basis by which we may evaluate the number of bits of any converter architecture (resistor-ladder, delta-sigma, etc.).

2 Delta Sigma Conversion Revealed

- A **delta sigma** D/A converter "transforms" (i.e. requantizes) an N-bit PCM signal into a 1-bit signal.
- Why requantize to a lower resolution? Because a 1-bit output is extremely easy to implement in hardware and there are ways to make that one-bit output have the SNR of an N-bit converter.
- How do you get an N-bit-to-1-bit quantizer, which would normally only produce a $6 \cdot 1 = 6$ dB SNR, to produce the required $6N$ dB SNR? *By using* ***oversampling*** *and* ***noise-shaping*** *to modify the 1-bit output.*

3 Oversampling

- Quantization noise is assumed white and uniformly-distributed with a total power of $q^2/12$, where q is the quantization step-size.
- NOTE: **The total quantization noise power does NOT depend on the sample rate!!!**
- Quantization noise modeled as a noise source added to the signal:

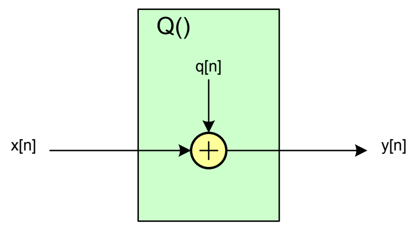

Figure 1: Quantizer Model

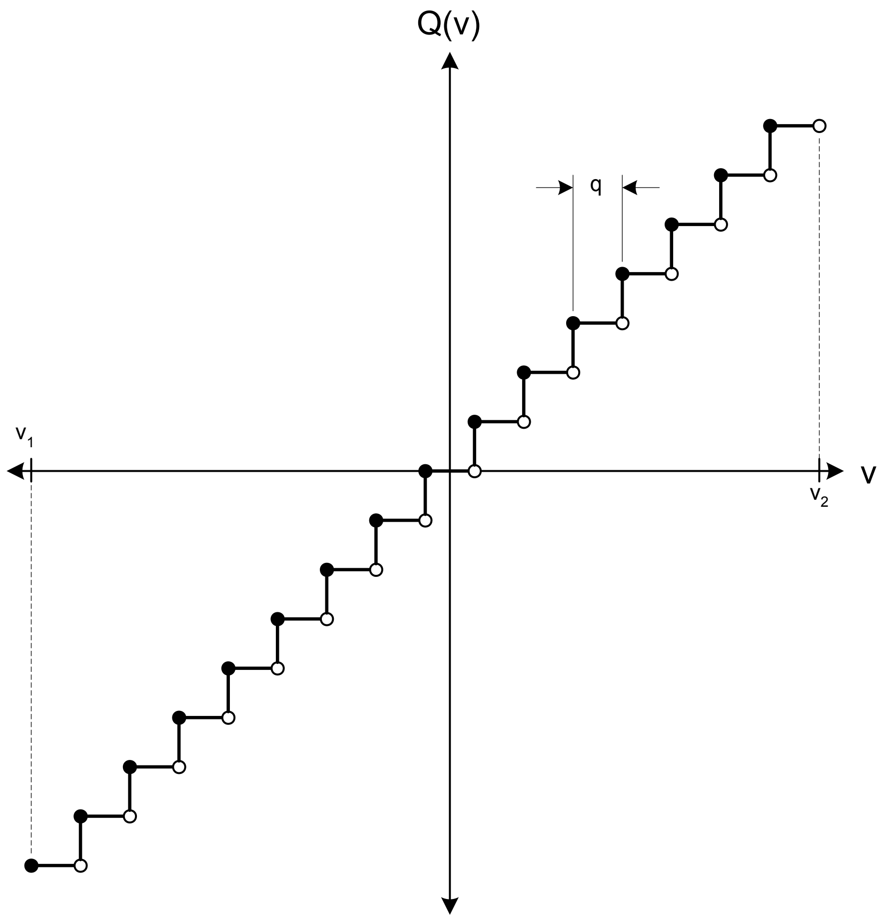

Figure 2: Quantizer Transfer Function

The "in-band" quantization noise power can be reduced by sampling at a rate higher than Nyquist.

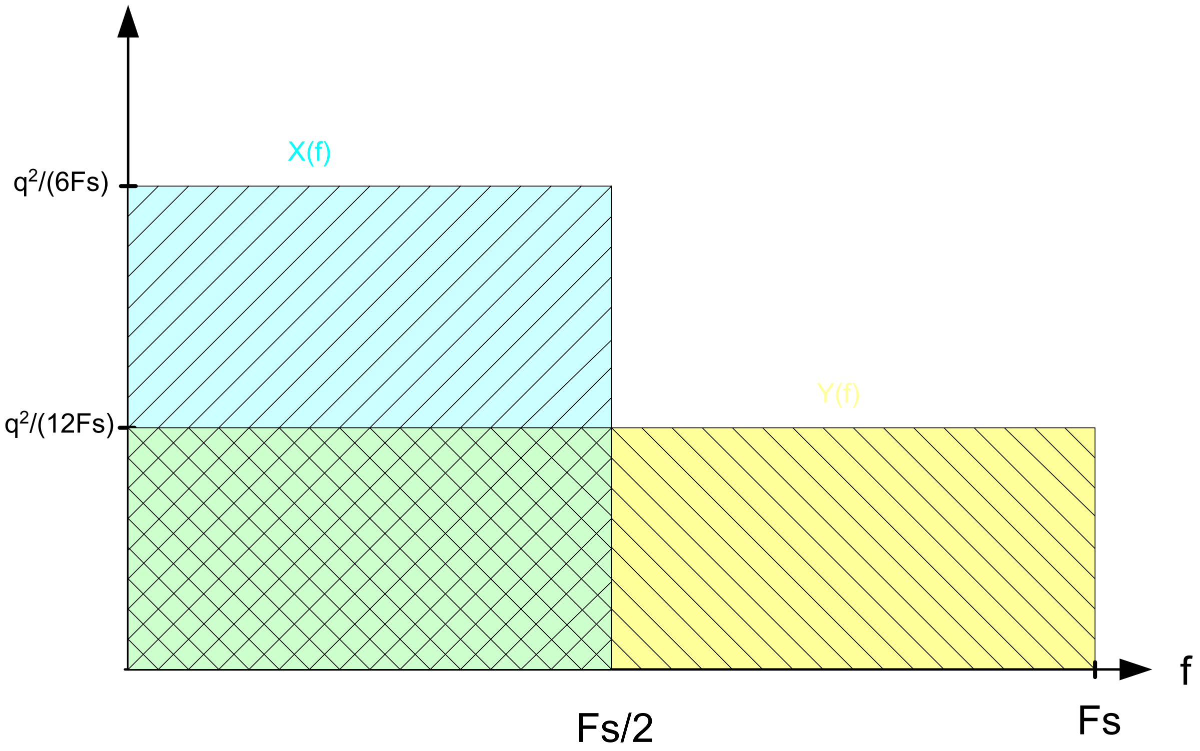

Figure 3: 2× Oversampled Quantization Noise Spectrum

Since the total in-band noise power is reduced, the number of "effective" bits is increased from the actual bits according to the relationship

$$M = 4^K,$$

where M is the oversampling factor and K is the number of extra bits.

Integer oversampling ratios are performed by using an interpolator:

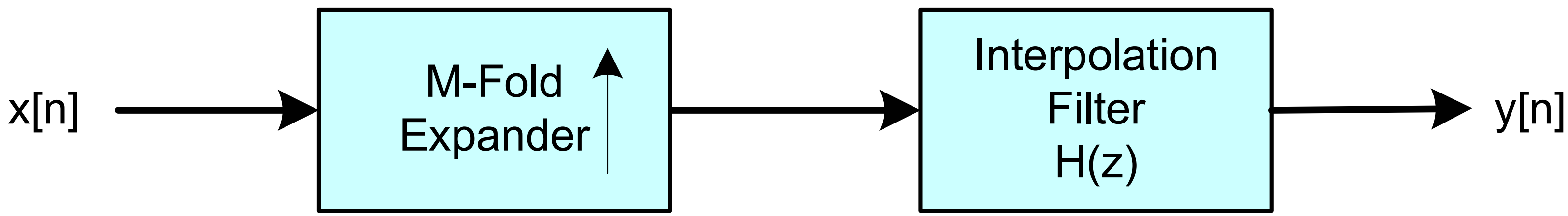

Figure 4: Interpolator Block Diagram

Oversampling alone is an inefficient way to obtain extra bits of resolution. A gain of even a few bits would require astronomical oversampling ratios! We must use the additional technique of *noise-shaping* to make a 1-bit converter feasible.

4 Noise-Shaping

Shapes the oversampled quantization noise spectrum so that less noise is in-band:

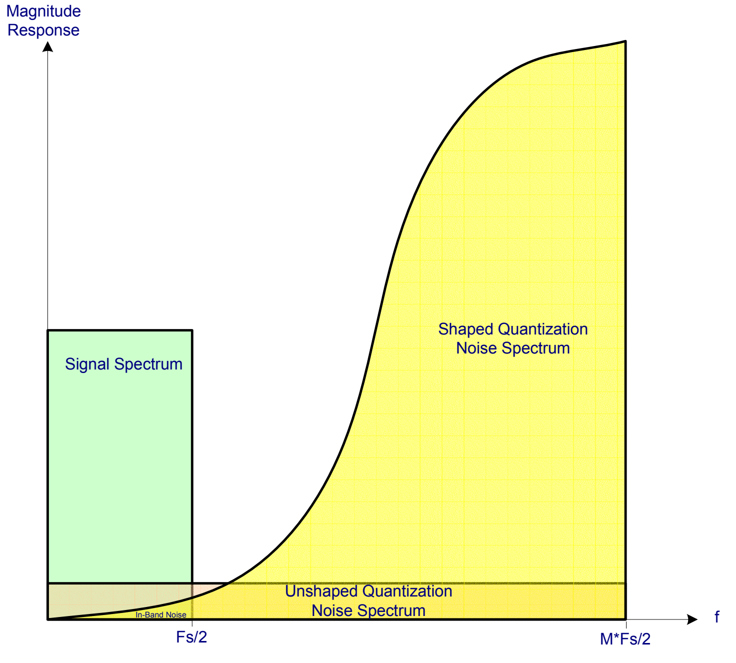

Figure 5: Typical Noise-Shaped Spectrum

Noise-shaping is accomplished by placing feedback around the quantizer:

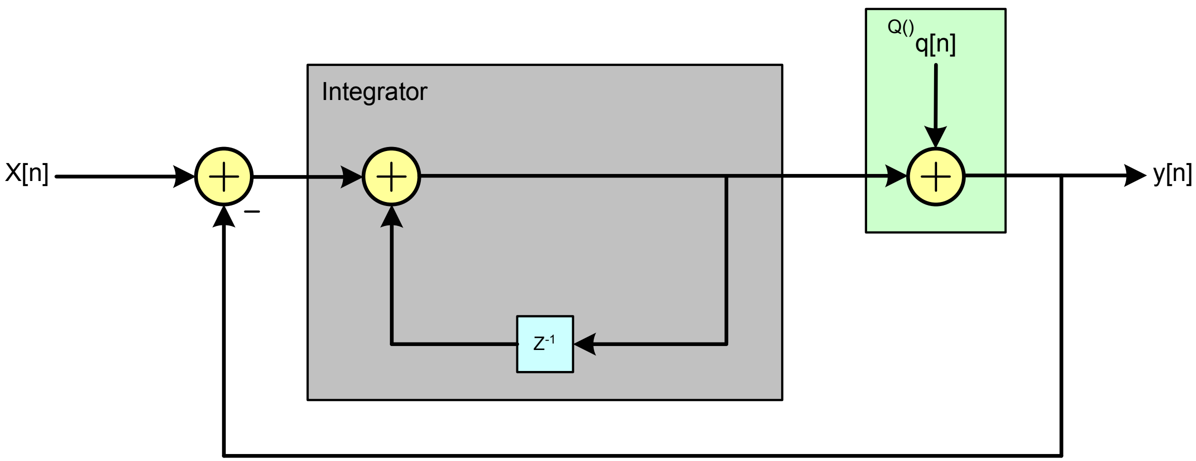

Figure 6: Classic First-Order Noise-Shaper

The transfer function of figure 6 is derived as follows:

$$\begin{aligned}
W(z) &= X(z) - z^{-1}Y(z) \\
\Sigma(z) &= W(z) + z^{-1}\Sigma(z) \Longrightarrow \Sigma(z) = \frac{W(z)}{1 - z^{-1}} \\
Y(z) &= \Sigma(z) + Q(z) = \frac{W(z)}{1 - z^{-1}} + Q(z) \\
(1 - z^{-1})Y(z) &= W(z) + (1 - z^{-1})Q(z) \\
&= X(z) - z^{-1}Y(z) + (1 - z^{-1})Q(z) \\
Y(z) &= X(z) + (1 - z^{-1})Q(z) \qquad (1)
\end{aligned}$$

It is clear from equation 1 that the signal $X(z)$ passes through unmodified while the quantization noise $Q(z)$ is modified by the term $1 - z^{-1}$. In delta-sigma modulator terminology this quantization noise coefficient is referred to as the *noise transfer function* [2], or NTF, denoted $N(z)$. Thus $N(z) = 1 - z^{-1}$.

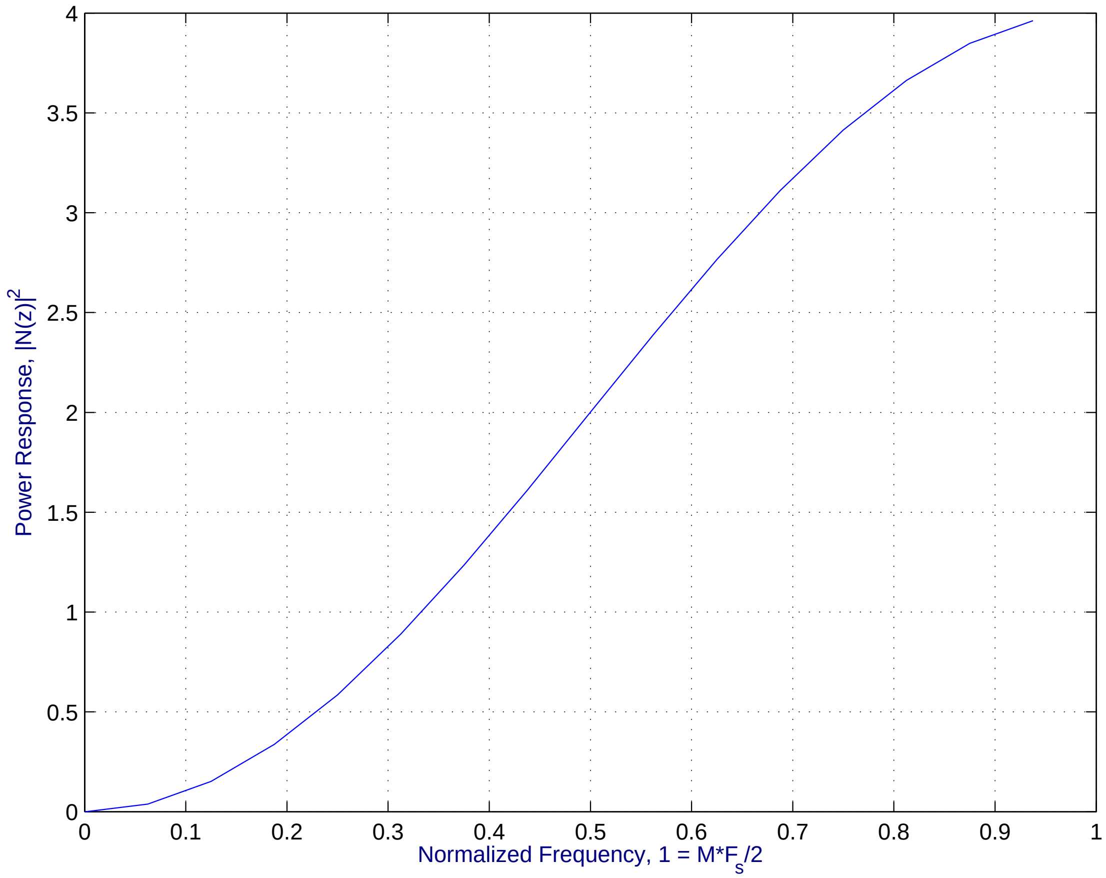

Figure 7: Noise Transfer Function Power Response of a First-Order Modulator

The noise-shaping can be made stronger by embedding integrator loops:

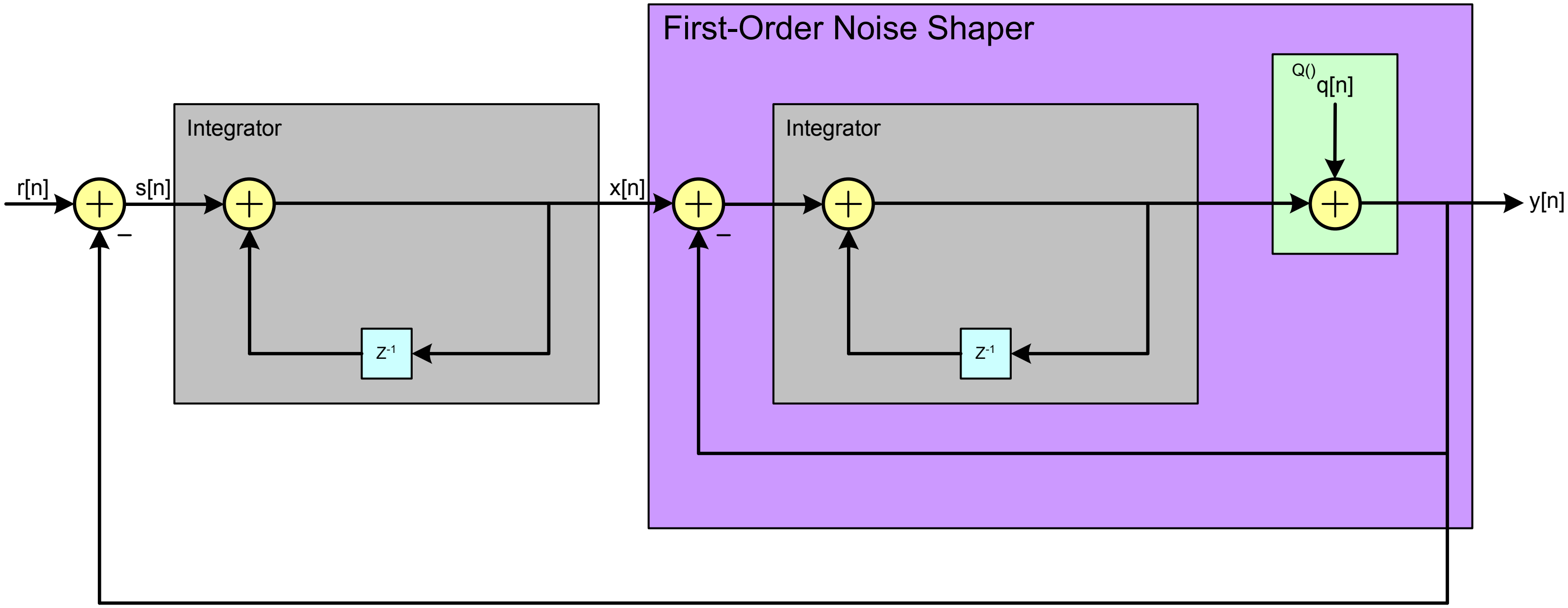

Figure 8: Second-Order Delta-Sigma Modulator

- The number of embeddings is termed the *order* of the modulator. An Lth-order modulator has NTF

$$N(z) = (1 - z^{-1})^L.$$

- It can be shown [3] that the in-band quantization noise power relative to the maximum signal power as a function of oversampling ratio M and modulator order L is

$$\frac{6L + 3}{2\pi^{2L}} M^{2L+1}.$$

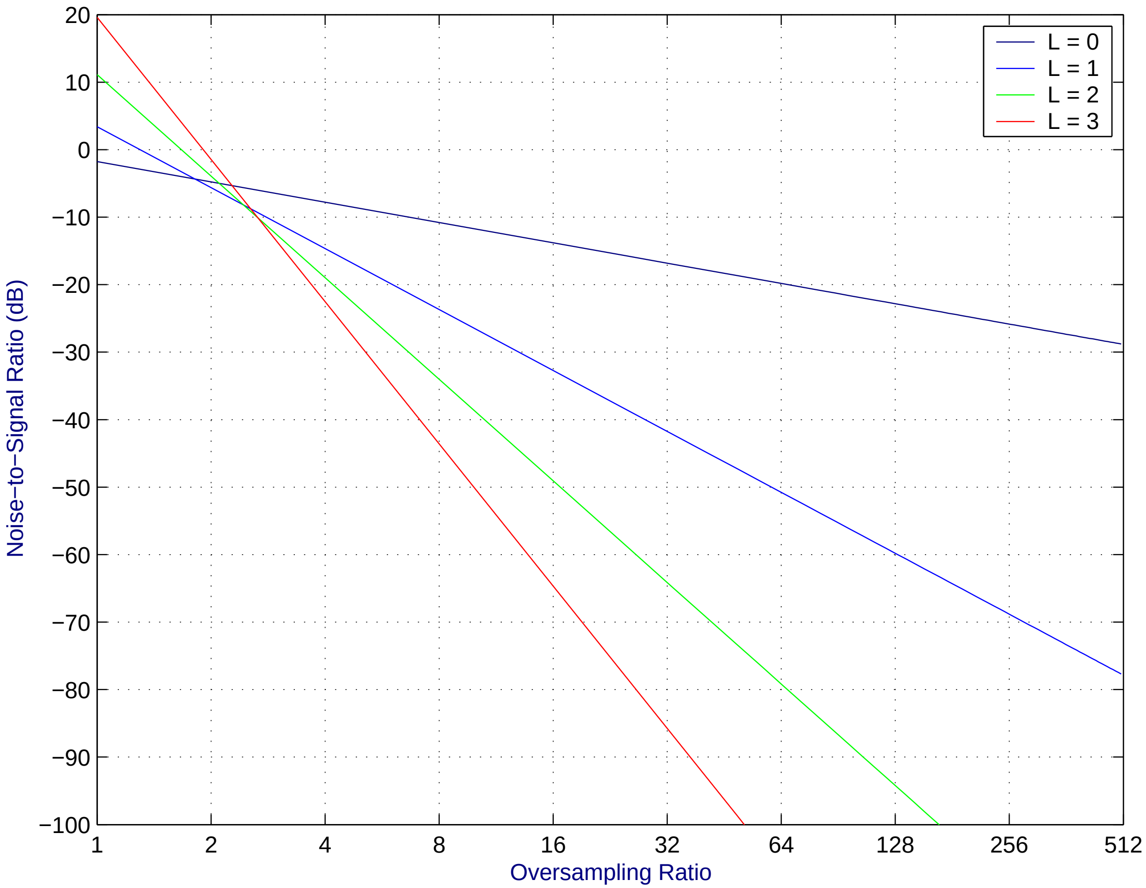

Figure 9: Ratio of In-Band Quantization Noise Power To Signal Power versus Oversampling Ratio and Modulator Order L

5 Alternate Modulator Architecture

$$Y(z) = X(z) + (1 - z^{-1}H(z))Q(z). \quad (2)$$

To be equivalent with the classic architecture, $H(z) = z - zG(z)$. Is $H(z)$ realizable???

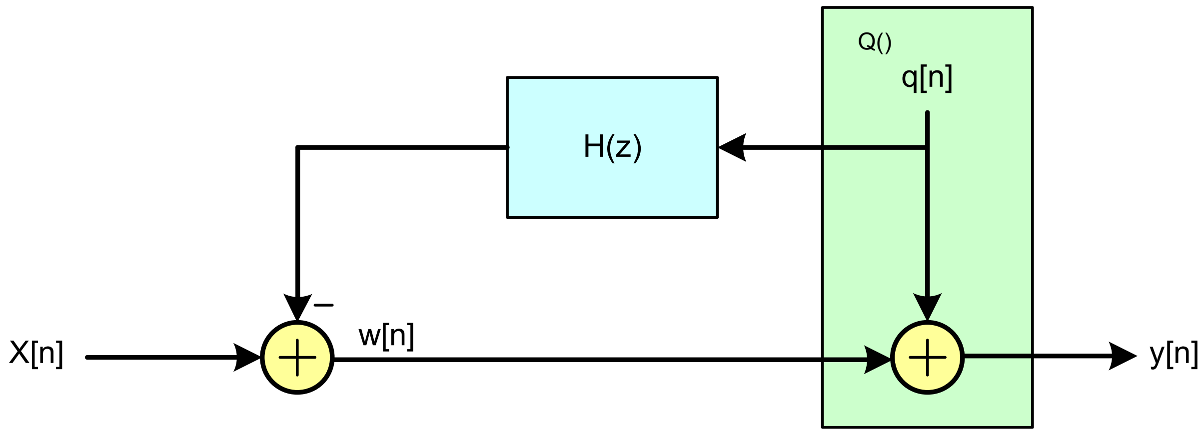

Figure 10: Alternate Delta-Sigma Modulator Architecture

Add dither to get rid of "birdies:"

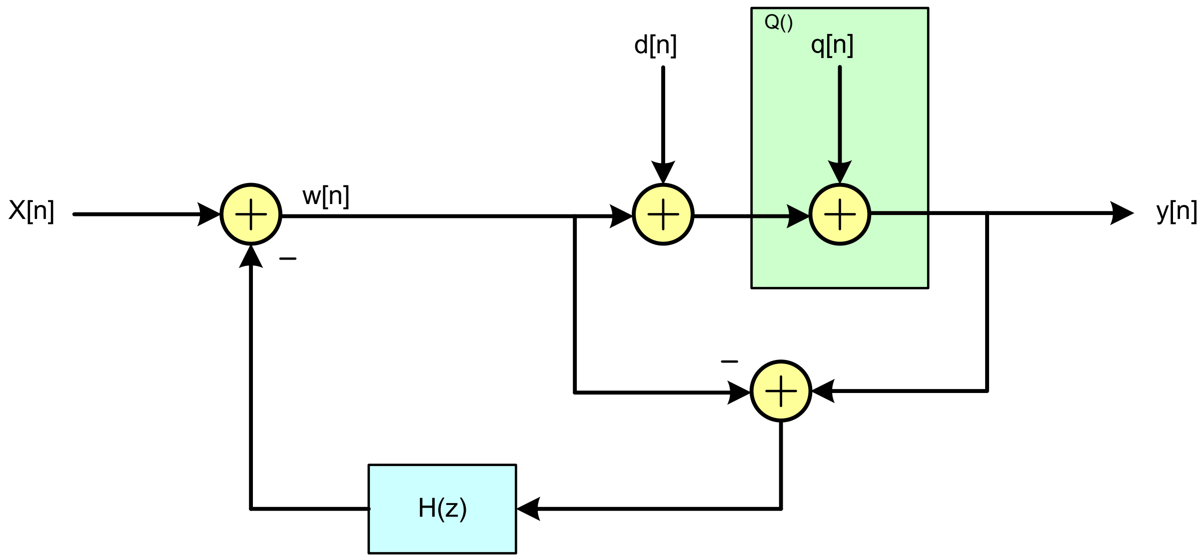

Figure 11: Delta Sigma Modulator with Dither

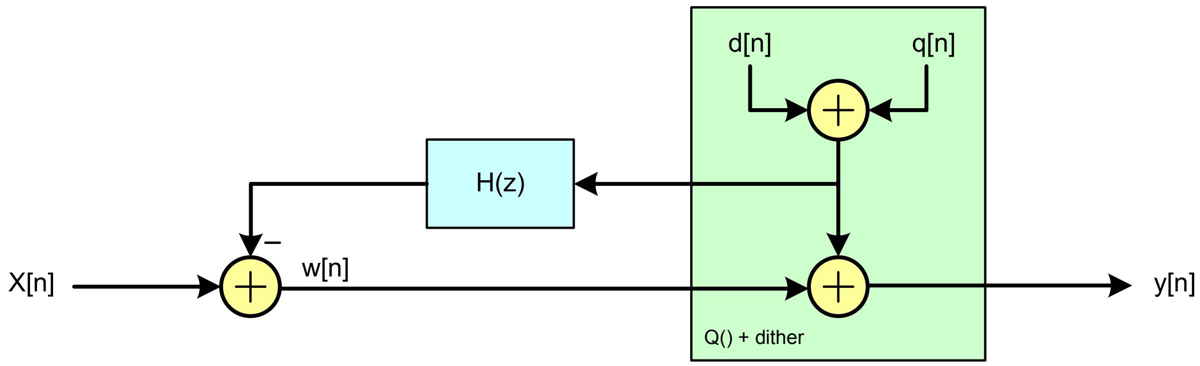

Figure 12: Equivalent Dithered Modulator

6 Psychoacoustic Noise-Shaping

- The alternate architecture admits any NTF of the form

$$N(z) = 1 - z^{-1}H(z).$$

- The classic Lth-order modulator NTF contains L zeros at $z = 1$ (DC),

$$N(z) = \frac{(z-1)^L}{z^L}.$$

- When L is even we can use conjugate pairs to place the zeros at any $L/2$ frequencies on the unit circle.

Example: For $L = 2$, we can place the zero at any frequency f, $0 \leq f \leq MF_s/2$:

$$N(z) = \frac{z^2 - 2\cos(\pi \frac{f}{MF_s}) + 1}{z^2}.$$

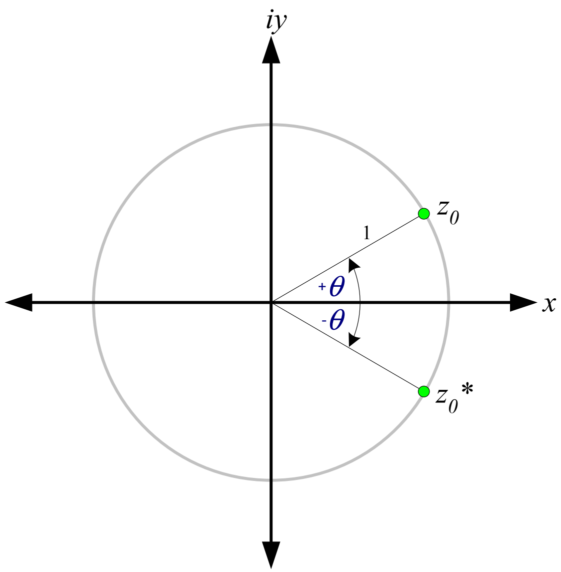

Figure 13: Zeros for Psychoacoustic Noise-Shaping, $\theta = \pi \frac{f}{MF_s}$.

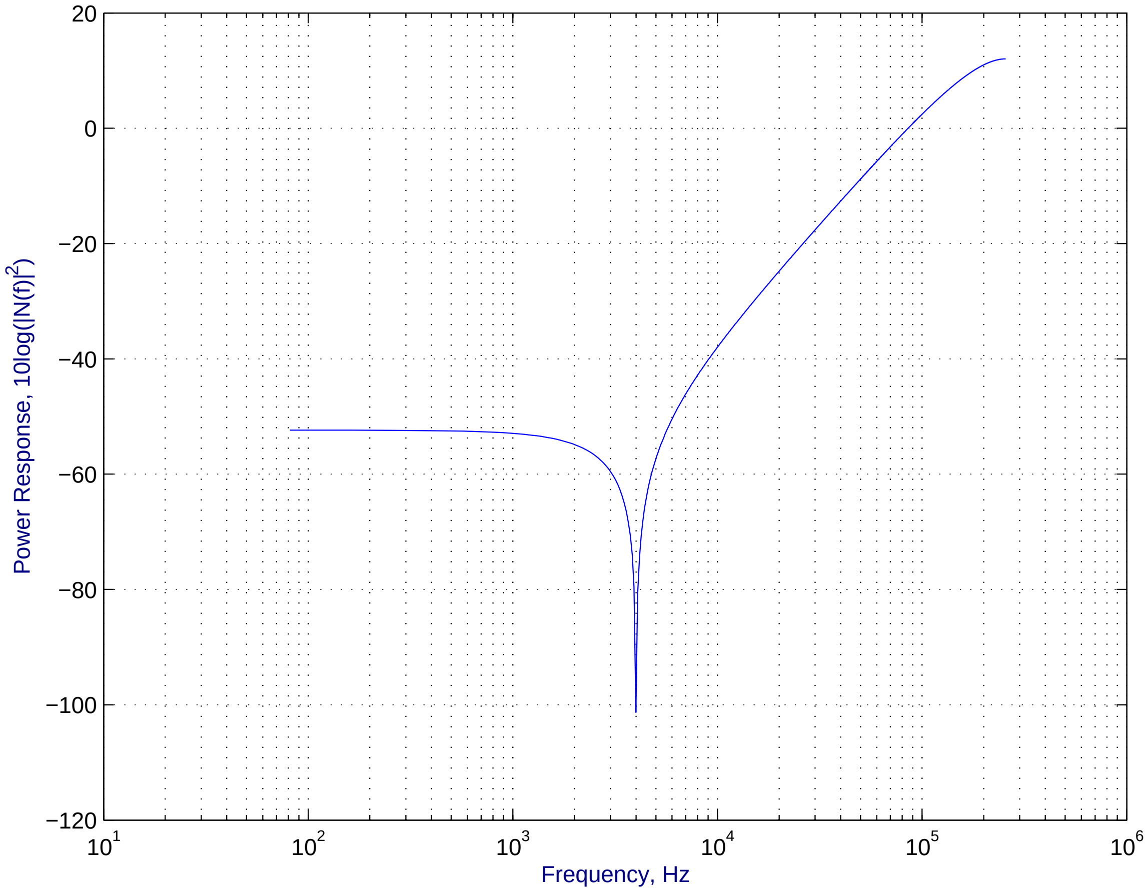

Figure 14: NTF Power Response $|N(f)|^2$ of Psychoacoustically Noise-Shaped Modulator with $f = 4$ kHz

7 The Complete Modulator

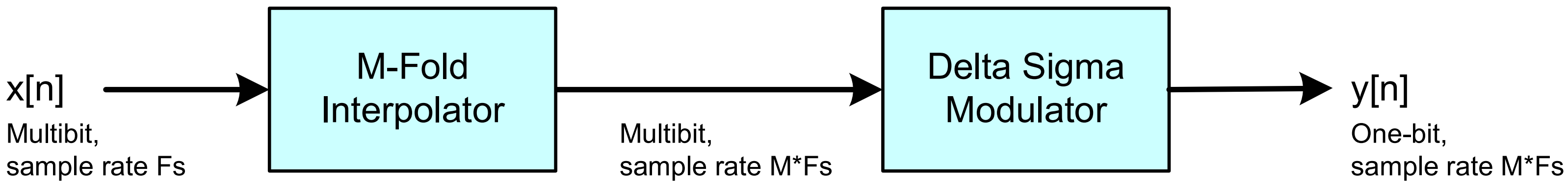

Figure 15: Delta Sigma D/A Converter Block Diagram

8 References

References

[1] Richard G. Lyons. *Understanding Digital Signal Processing*. Prentice Hall, second edition, 2004.

[2] Steven R. Norsworthy, Richard Schreier, and Gabor C. Temes. *Delta-Sigma Data Converters: Theory, Design, and Simulation*. IEEE Press, 1997.

[3] David Johns and Ken Martin. *Analog Integrated Circuit Design*. Wiley Publishers, 1997.