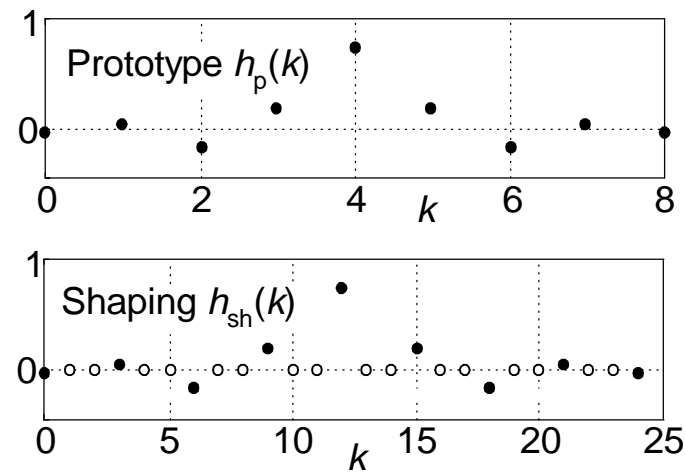
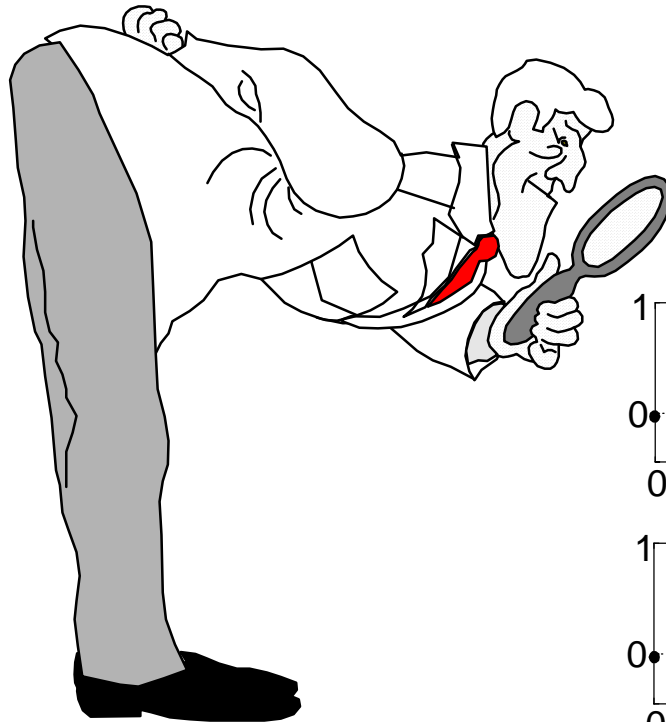


2004 COMP.DSP Conference; Cannon Falls, MN, July 29-30, 2004

Interpolated Lowpass FIR Filters

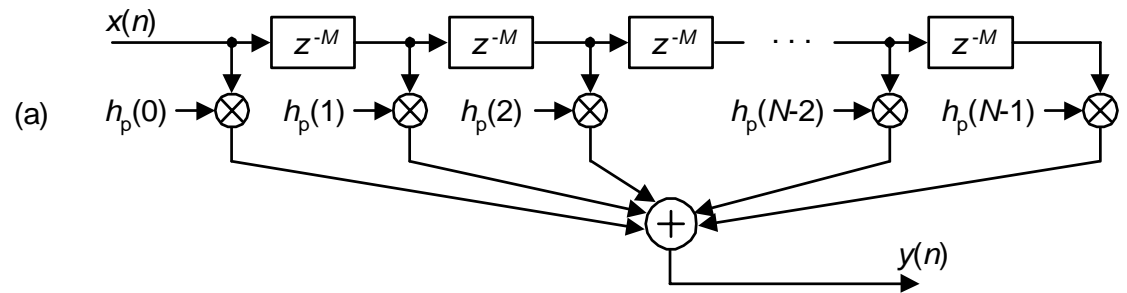
Speaker: **Richard Lyons**
Besser Associates
E-mail: r.lyons@ieee.com



Interpolated FIR Filters

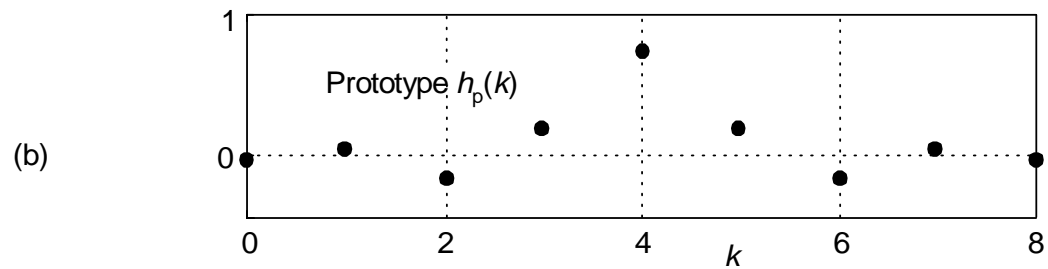
- ▶ ***Interpolated FIR filters* are used to build narrowband lowpass FIR filters,**
 - possibly more computationally efficient than traditional Parks-McClellan-designed FIR filters.

- ▶ **Interpolated FIR (IFIR) filters are based upon the behavior of an N -tap nonrecursive linear-phase FIR filter,**
 - when each of its single-unit delays are replaced with M -unit delays,
 - where M is an integer.

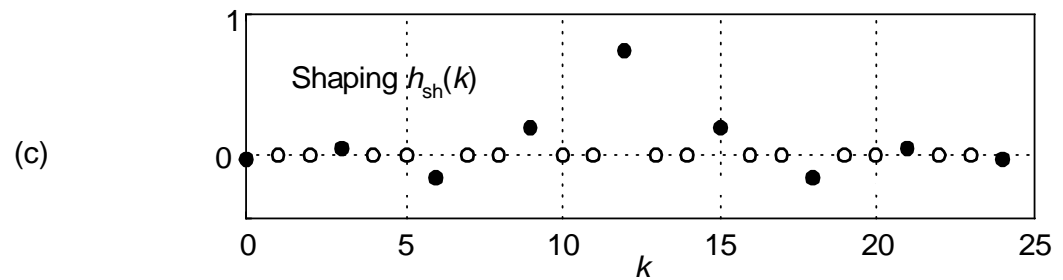


For example:

$h_p(k)$ impulse response of a 9-tap FIR *prototype* filter.



$h_{sh}(k)$ impulse response of an expanded FIR filter, where $M = 3$. We the expanded filter the *shaping* filter.



- ▶ **Prototype FIR filter's transfer function as**

$$H_p(z) = \sum_{k=0}^{N_p-1} h_p(k)z^{-k}$$

- where N_p is the length of $h_p(k)$, and k is the filter coefficient index.

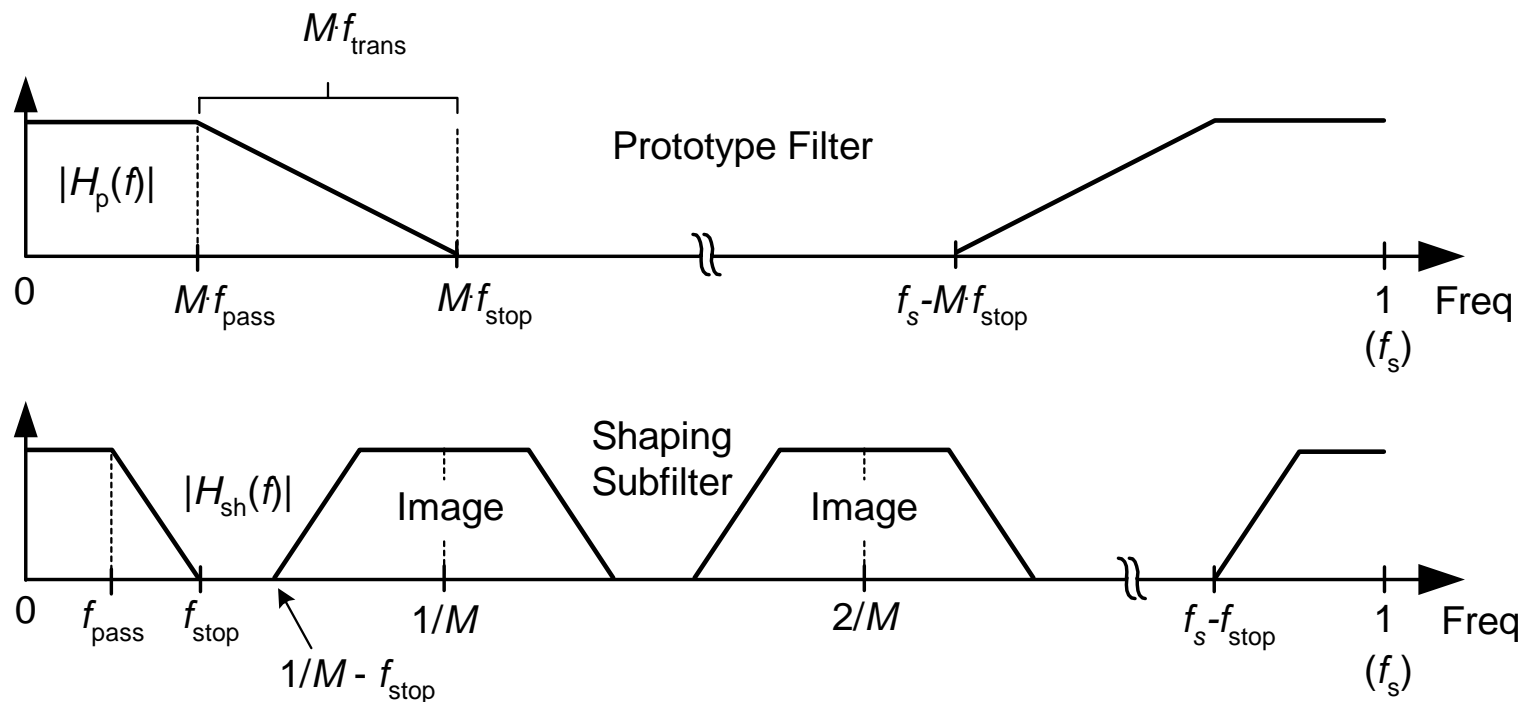
- ▶ **Transfer function of a general shaping FIR filter [z in $H_p(z)$ replaced with z^M] is**

$$H_{sh}(z) = \sum_{k=0}^{N_p-1} h_p(k)z^{-kM}.$$

- ▶ **If the number of coefficients in the prototype filter is N_p ,**
 - **expanded impulse response length of shaping filter is**

$$K_{sh} = M(N_p - 1) + 1.$$

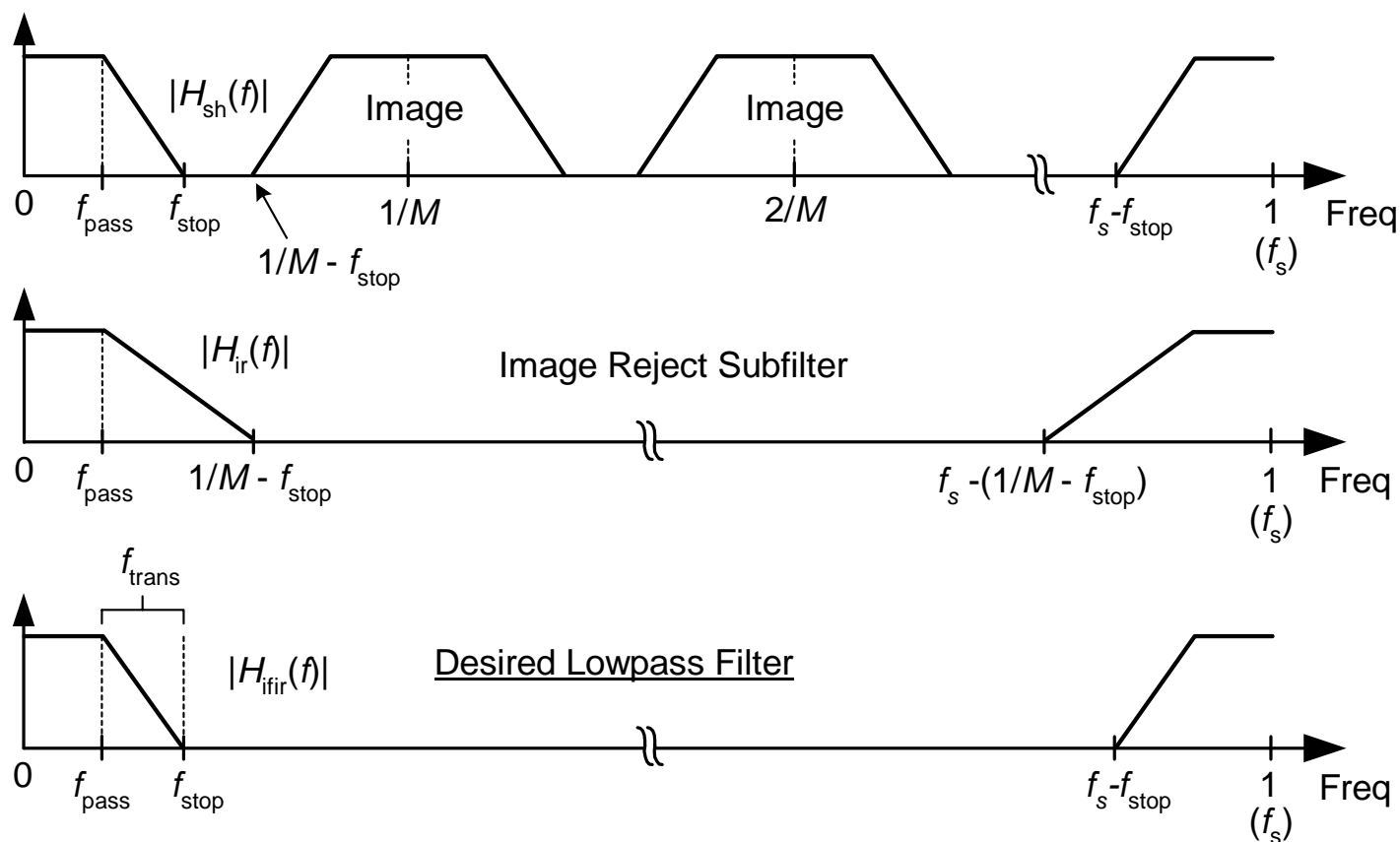
- ▶ An M -fold expansion of the impulse response causes an M -fold compression (and repetition) of $|H_p(f)|$ frequency magnitude response.
- ▶ There are M repetitive passbands in $|H_{sh}(f)|$,
 - centered at integer multiples of $1/M$ (f_s/M Hz),
 - called *images*.



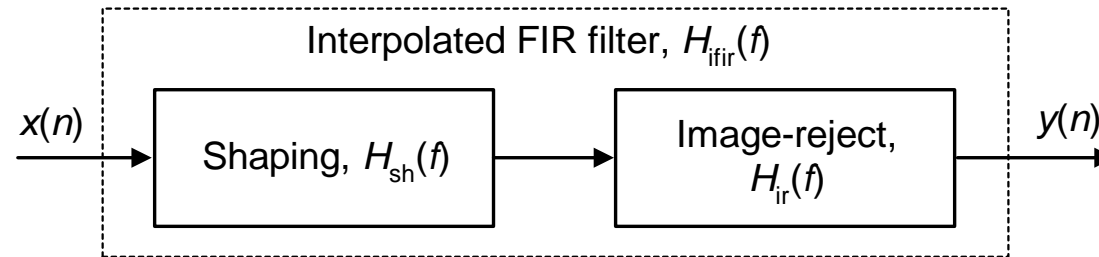
- ▶ Next, we follow the shaping subfilter with a lowpass *image-reject* subfilter,
 - whose task is to attenuate the *image* passbands,

- ▶ The resultant $|H_{\text{ifir}}(f)|$ frequency magnitude response is, of course, the product

$$|H_{\text{ifir}}(f)| = |H_{\text{sh}}(f)| |H_{\text{ir}}(f)|.$$

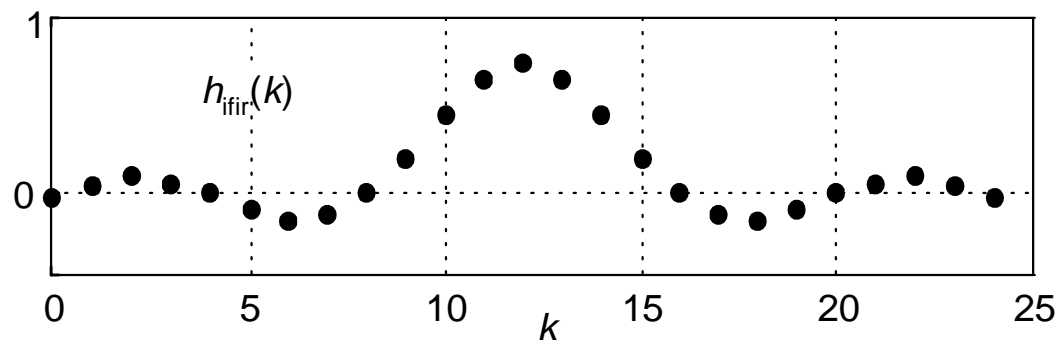


Cascaded subfilters
is called an
**Interpolated FIR
(IFIR) filter.**



IFIR filter
interpolated
impulse response.

(Interpolated
version of $h_p(k)$.)



► Original desired lowpass filter's passband width is f_{pass} ,

- its stopband begins at f_{stop} , and

- Its transition region width is $f_{\text{trans}} = f_{\text{stop}} - f_{\text{pass}}$,

► Then the prototype subfilter's normalized frequency parameters are defined as

$$f_{\text{p-pass}} = Mf_{\text{pass}}, \quad f_{\text{p-stop}} = Mf_{\text{stop}}, \quad \text{and} \quad f_{\text{p-trans}} = Mf_{\text{trans}} = M(f_{\text{stop}} - f_{\text{pass}}).$$

- ▶ **The image-reject subfilter's frequency parameters are**

$$f_{\text{ir-pass}} = f_{\text{pass}}, \text{ and } f_{\text{ir-stop}} = \frac{1}{M} - f_{\text{stop}}.$$

- ▶ **Stopband attenuations of the prototype filter and image-reject subfilter are identical,**
 - **set equal to the desired IFIR filter stopband attenuation.**

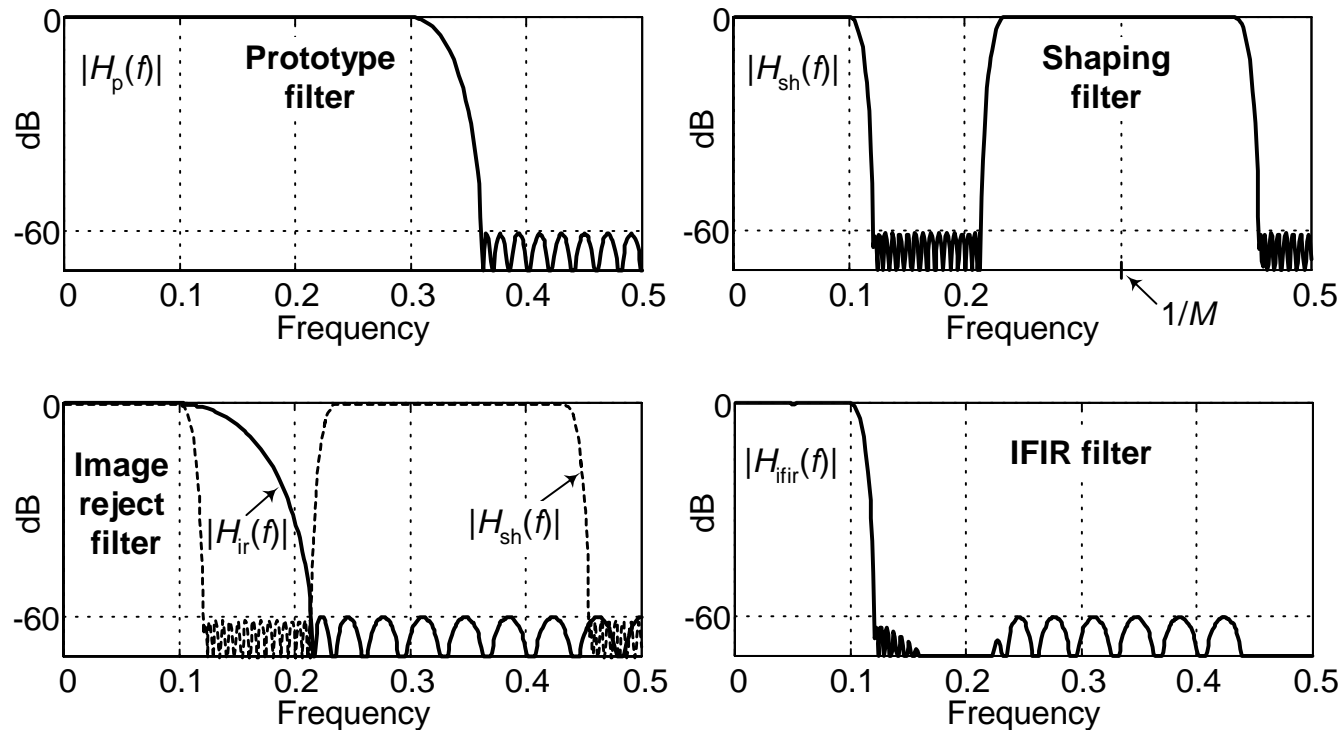
- ▶ **Let's look at a design example:**

- ▶ **Consider the design of a *desired* linear-phase FIR filter:**

- **normalized passband width is $f_{\text{pass}} = 0.1$,**
- **passband ripple is 0.1 dB, (peak-peak)**
- **transition region width is $f_{\text{trans}} = 0.02$, and**
- **stopband attenuation is 60 dB.**

- ▶ **Expansion factor of $M = 3$.**

► Here's what we have:



► Satisfying the original desired filter specifications would

- require a traditional single-stage FIR filter with $N_{\text{tfir}} = 137$ taps,
- 'tfir' subscript means *traditional FIR*.

► Shape of $|H_{\text{ifir}}(f)|$ determined by $|H_{\text{sh}}(f)|$ "shaping subfilter".

- ▶ **IFIR's shaping and the image-reject subfilters require $N_p = 45$ and $N_{ir} = 25$ taps respectively,**
 - **for a total of $N_{ifir} = 70$ taps.**

- ▶ **We define the percent reduction in computational workload as**

$$\% \text{ computation reduction} = 100 \frac{N_{\text{tfir}} - N_p - N_{ir}}{N_{\text{tfir}}}. \quad (1)$$

- ▶ **IFIR filter computational workload reduction:**

$$\% \text{ computational reduction} = 100 \frac{137 - 70}{137} = 49\%. \quad \text{😊}$$

Choosing the Optimum Expansion Factor M

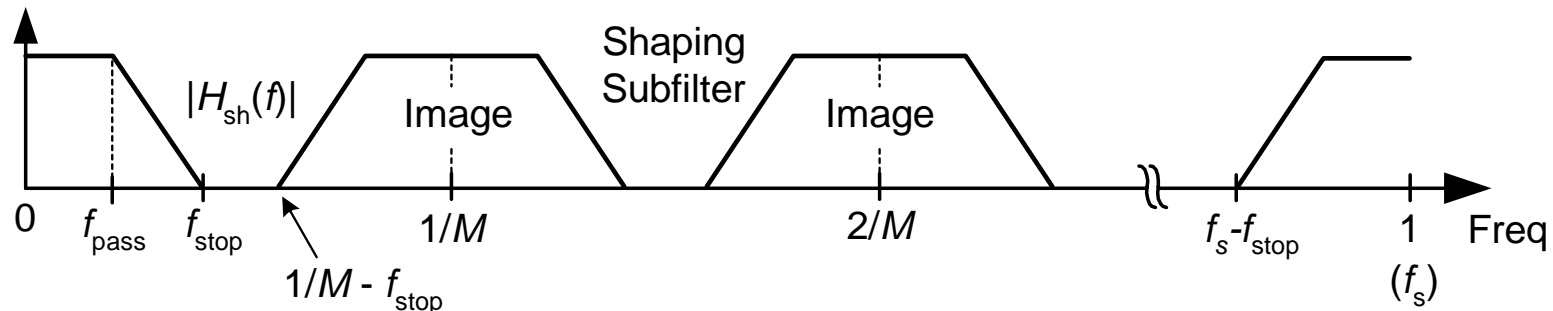
- ▶ Expansion factor M has a profound effect on the computational efficiency of IFIR filters.
- ▶ To show this, consider other values of expansion factor M .

Expansion factor M	Number of taps			Computation reduction
	$h_{sh}(k)$	$h_{ir}(k)$	IFIR total	
2	69	8	77	43%
3	45	25	70	49%
4	35	95	130	8%

- ▶ As so often happens in signal processing designs, there is a trade off to be made.
 - Smaller M , reduced frequency compression in $H_{sh}(f)$, increases necessary N_p taps,
 - Larger M , reduces transition region width of $H_{ir}(f)$, increases necessary N_{ir} taps.

► As indicated in the following figure,

- $\max M$ is the largest integer satisfying $1/M - f_{\text{stop}} \geq f_{\text{stop}}$, (or $1/M \geq 2f_{\text{stop}}$),
- ensuring no passband image overlap.



► This yields an upper bound on M of

$$M_{\max} = \left\lfloor \frac{1}{2f_{\text{stop}}} \right\rfloor$$

- where $\lfloor x \rfloor$ indicates truncation of x to an integer.

► Thus the acceptable expansion factors are integers in the range $2 \leq M \leq M_{\max}$.

► For our above IFIR filter design example:

$$M_{\max} = \left\lfloor \frac{1}{2(0.1 + 0.02)} \right\rfloor = 4.$$

Estimating the Number of FIR Filter Taps

- ▶ To estimate the computation reduction of IFIR filters,
 - we need an algorithm to compute N_{tfir} ,
 - the number of taps, in a traditional nonrecursive FIR filter.
- ▶ A particularly simple expression for N_{tfir} is

$$N_{\text{tfir}} \approx \frac{\textit{Atten}}{22(f_{\text{stop}} - f_{\text{pass}})} \cdot \quad (2)$$

- Where *Atten* = stopband attenuation in dB

- ▶ Likewise, the number of taps in the prototype and image-reject subfilters are

$$N_{\text{p}} \approx \frac{\textit{Atten}}{22(M)(f_{\text{stop}} - f_{\text{pass}})} \cdot \text{ and} \quad (2')$$

$$N_{\text{ir}} \approx \frac{\textit{Atten}}{22(1/M - f_{\text{stop}} - f_{\text{pass}})} \cdot \quad (2'')$$

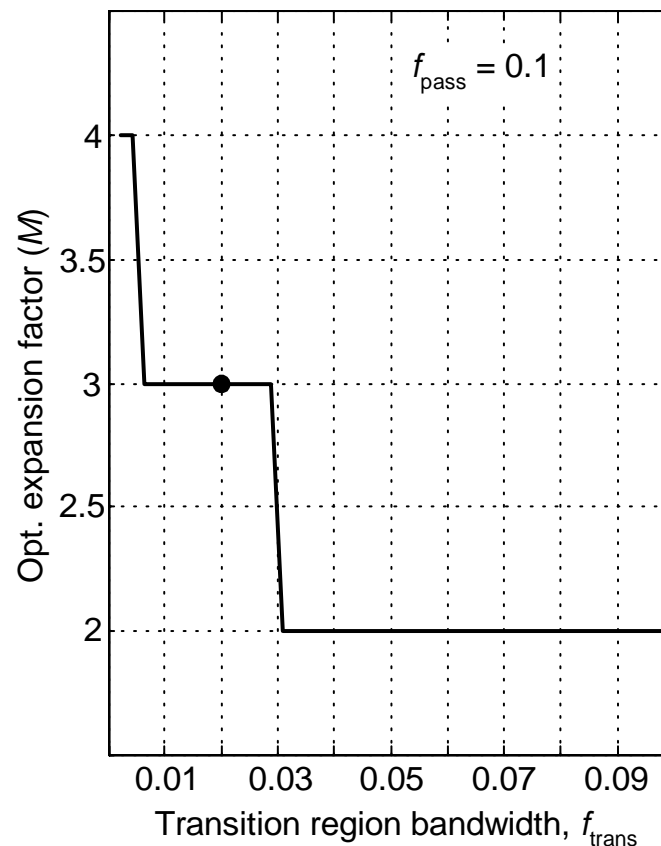
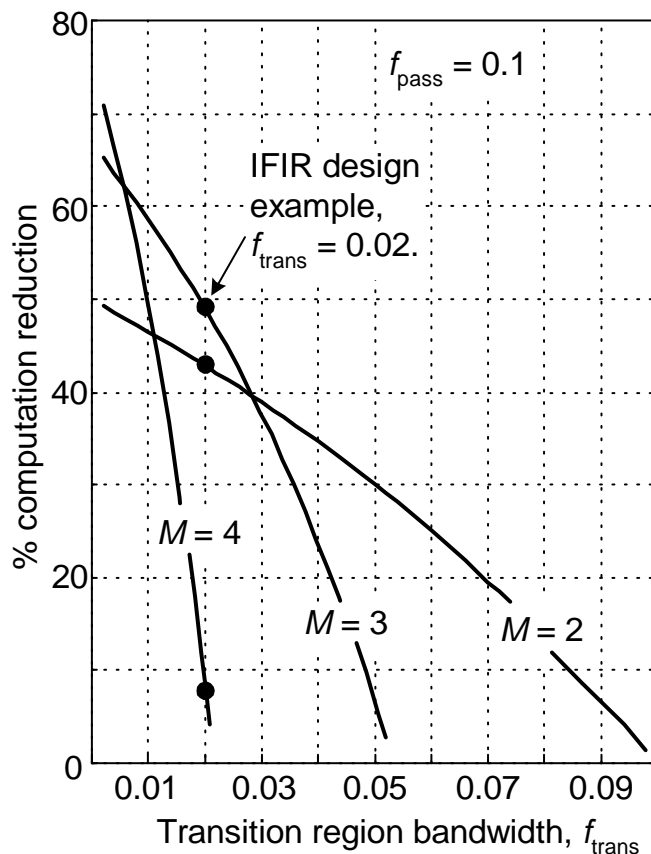
Modeling IFIR Filter Performance

- ▶ We want to model "% computation reduction" in terms of desired filter parameters.
- ▶ If we substitute the expressions from Eq. (2) into Eq. (1),
 - we can write the important IFIR filter design equation:

$$\% \text{ computation reduction} = 100 \left[\frac{M-1}{M} - \frac{Mf_{\text{trans}}}{1 - Mf_{\text{trans}} - 2Mf_{\text{pass}}} \right]. \quad (3)$$

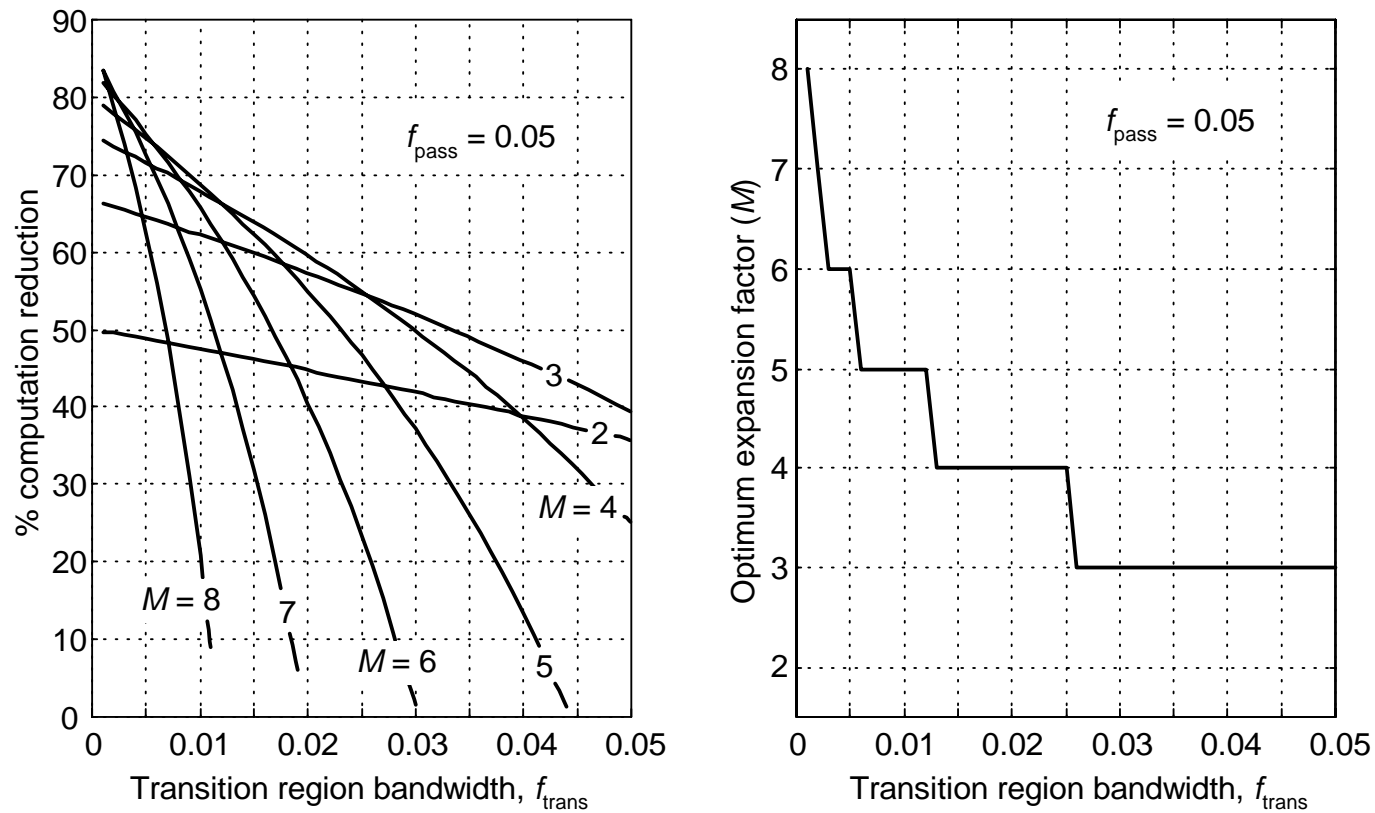
- where $f_{\text{trans}} = f_{\text{stop}} - f_{\text{pass}}$.

- ▶ Equation (3) is plotted below, for $f_{\text{pass}} = 0.1$
 - showing % computation reduction vs. f_{trans} .



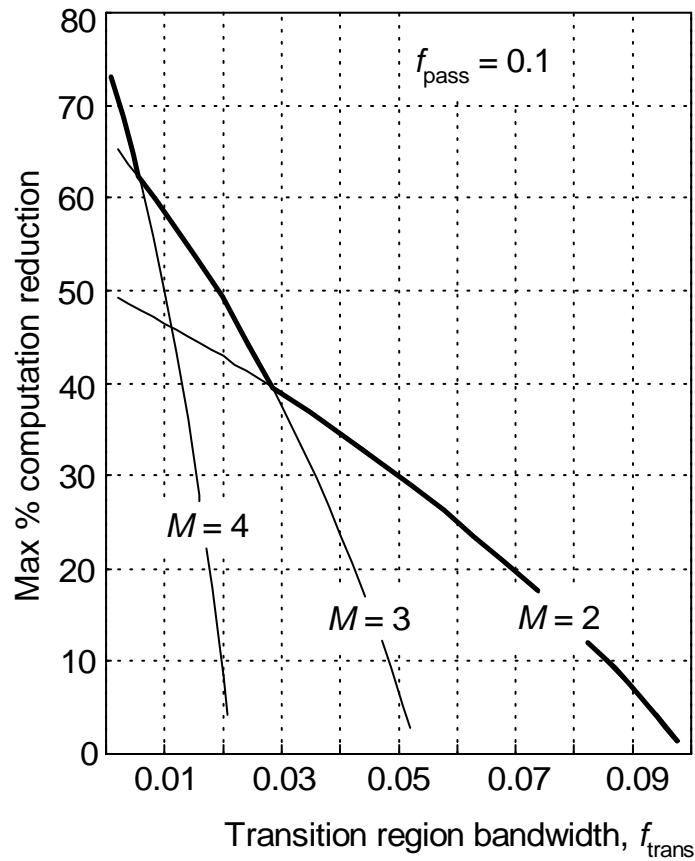
- ▶ When the transition region width is large, only a small M will avoid passband image overlap.
- ▶ At smaller transition region widths, larger expansion factors are possible.

► Here's IFIR filter performance when the $f_{\text{pass}} = 0.05$.

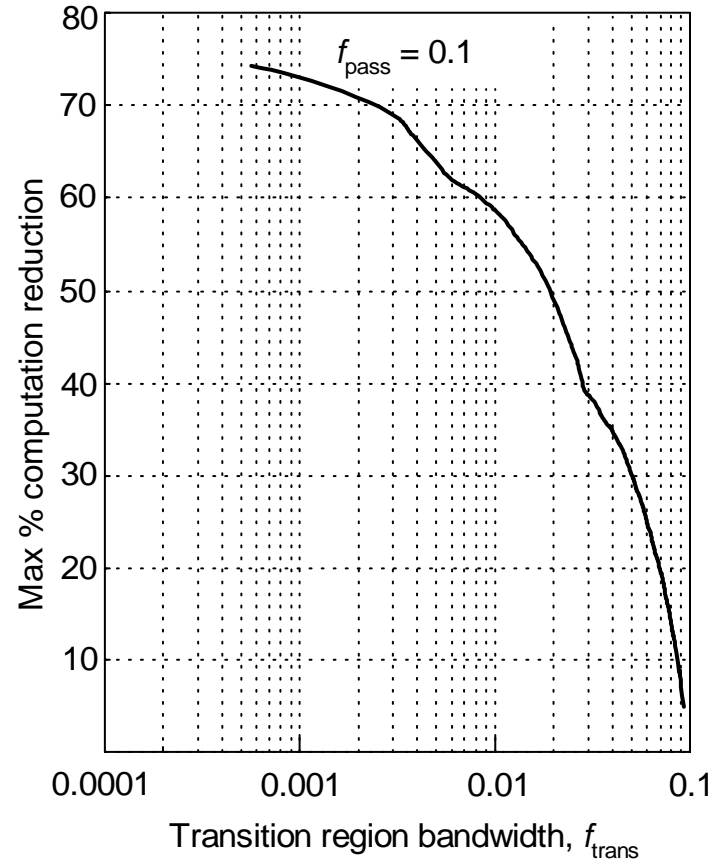


► As f_{trans} approaches zero, % computation reduction approaches $100(M-1)/M$.

- Here we plot max % computation reduction as a function of f_{trans} for $f_{\text{pass}} = 0.1$
 - on a logarithmic frequency axis.

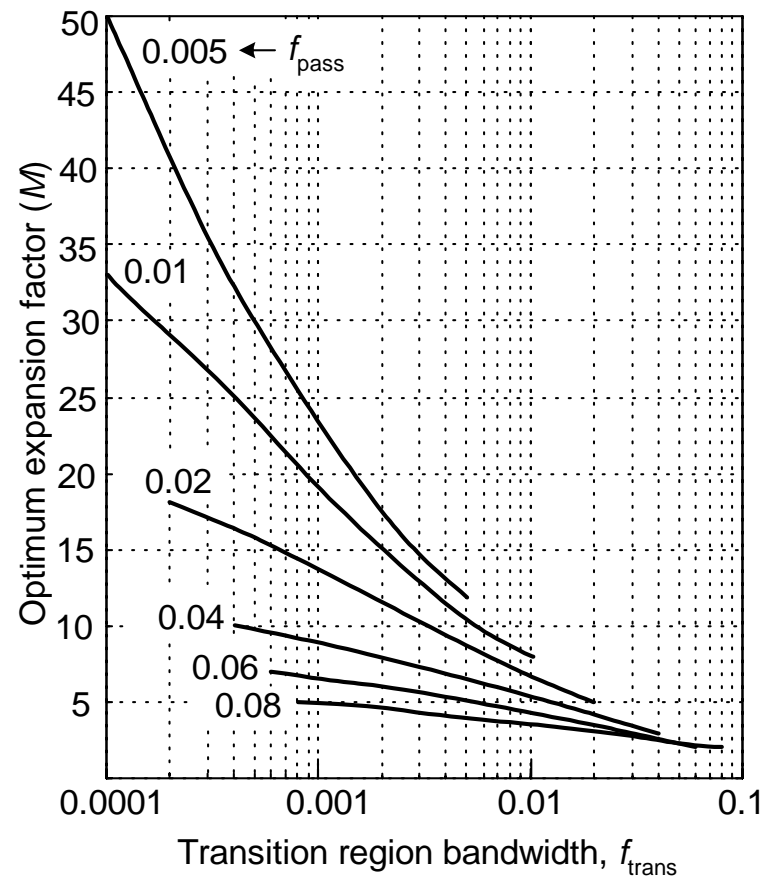
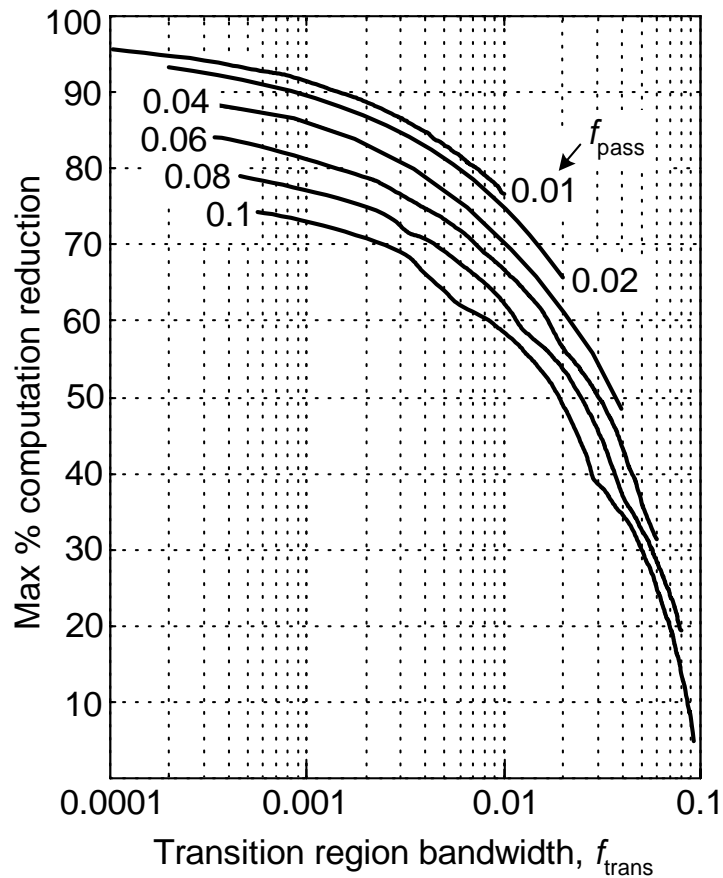


(a)



(b)

- ▶ Next, we include other f_{pass} curves to show max % computation reduction vs. f_{trans} ,
 - and optimum M used to compute the max % computation reduction curves.



- ▶ These are our **IFIR filter design curves**. 😊

IFIR Filter Implementation Issues

- ▶ Please resist the temptation to combine the two subfilters into a single filter
 - whose coefficients are the convolution of the subfilters' impulse responses.
 - With such a maneuver would we'd lose all computation reduction.
- ▶ When using programmable DSP chips, larger values of M require a larger block of hardware *data memory*, in the form of a *circular buffer*, be available for the shaping subfilter.
- ▶ The size of this data memory must be at least

$$K_{\text{sh}} = M(N_p - 1) + 1.$$

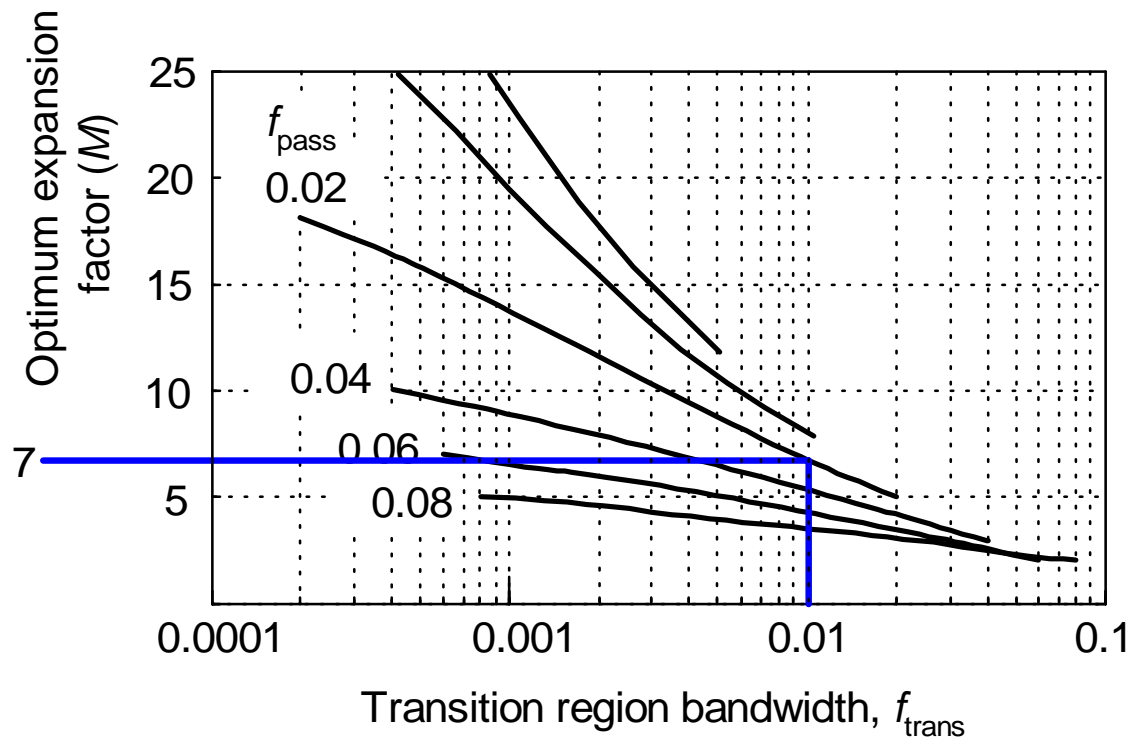
- ▶ When implementing an IFIR filter with a programmable DSP chip,
 - you must loop through the circular signal data buffer using an increment equal to M .
- ▶ If possible, use *folded* nonrecursive FIR structures,
 - to reduce the number of multiplications by a factor of two.

IFIR Filter Design Example

- ▶ **The design of practical lowpass IFIR filters is straightforward, and comprises four steps:**
 - **Define the desired lowpass filter performance requirements,**
 - **Determine a candidate value for the expansion factor M ,**
 - **Design and evaluate the shaping and image-reject subfilters, and**
 - **Investigate IFIR filter performance for alternate expansion factors near the initial M value.**

- ▶ **As a design example, we'll design a lowpass IFIR filter with:**
 - **$f_{\text{pass}} = 0.02$,**
 - **passband ripple of 0.5 dB (p-p),**
 - **$f_{\text{trans}} = 0.01$ (thus $f_{\text{stop}} = 0.03$), and**
 - **stopband attenuation = 50 dB.**

- ▶ First, we find the $f_{\text{trans}} = 0.01$ point on the abscissa of our design curve and
 - follow it up to the point where it intersects the $f_{\text{pass}} = 0.02$ curve.
 - This intersection indicates we should start our design with $M = 7$.



- ▶ With $M = 7$, we use our favorite traditional FIR filter design software to design a linear-phase prototype FIR filter with the following parameters:

$$f_{p\text{-pass}} = Mf_{\text{pass}} = 7(0.02) = 0.14,$$

$$\text{passband ripple} = (0.5)/2 \text{ dB} = 0.25 \text{ dB}, \quad (\text{rule of thumb})$$

$$f_{p\text{-stop}} = Mf_{\text{stop}} = 7(0.03) = 0.21, \text{ and}$$

$$\text{stopband attenuation} = 50 \text{ dB}.$$

- ▶ Such a prototype FIR filter will have $N_p = 33$ taps and, with $M = 7$,
 - shaping subfilter has an impulse response length of $K_{\text{sh}} = 225$ samples.

- ▶ Next, we design an image-reject subfilter having the following parameters:

$$f_{\text{ir-pass}} = f_{\text{pass}} = 0.02,$$

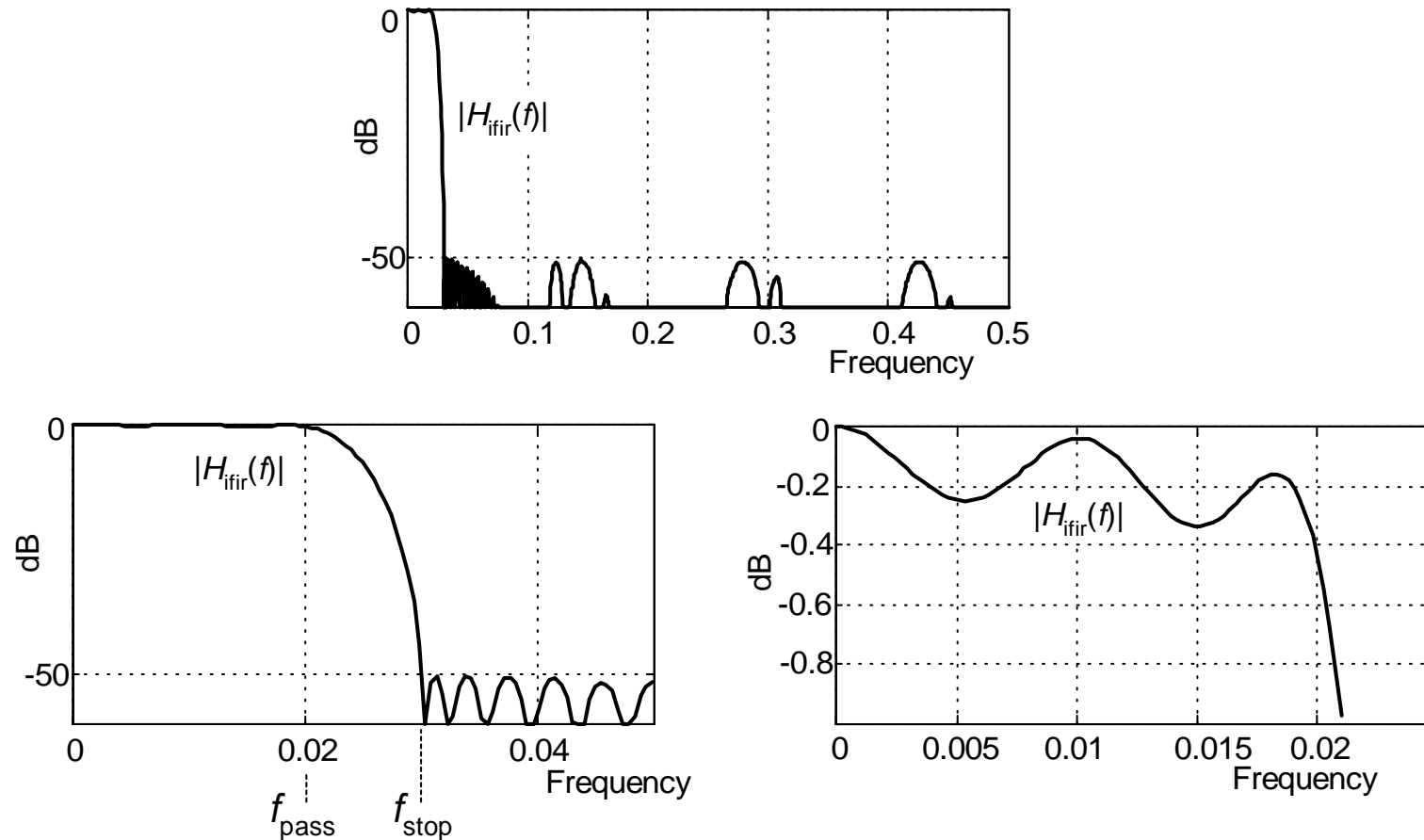
$$\text{passband ripple} = (0.5)/2 \text{ dB} = 0.25 \text{ dB},$$

$$f_{\text{ir-stop}} = \frac{1}{M} f_{\text{stop}} = 1/7 - 0.03 = 0.113, \text{ and}$$

$$\text{stopband attenuation} = 50 \text{ dB}.$$

- ▶ This image-reject subfilter will have $N_{\text{ir}} = 27$ taps.

- ▶ **Cascaded image-reject and shaping subfilters require 60 multiplications per output sample.**
 - **IFIR filter frequency magnitude response is shown below.**



- ▶ **A traditional FIR filter requires roughly $N_{\text{tfir}} = 240$ taps.**

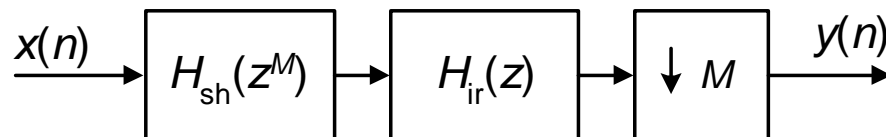
- ▶ **Computational workload reduction is $100 \times (240 - 60) / 240 = 75\%$! 😊**
 - **Final IFIR filter design step is to sit back and enjoy a job well done.**

- ▶ **Further modeling, using alternate expansion factors, yields the following table.**

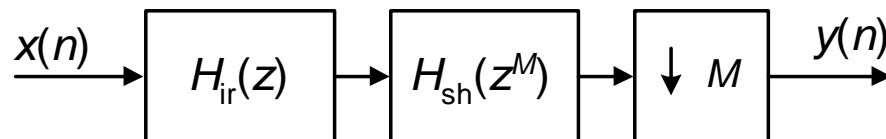
Expansion factor M	Number of taps			K_{sh} data storage	Computation reduction:
	$h_{sh}(k)$	$h_{ir}(k)$	IFIR total		
3	76	8	84	226	65%
4	58	12	70	229	71%
5	46	17	63	226	74%
6	39	22	61	229	75%
7	33	27	60	225	75%
8	29	33	62	225	74%
9	26	41	67	226	72%
10	24	49	73	231	70%
11	21	60	81	221	66%

IFIR Filters With Sample Rate Conversion (SRC)

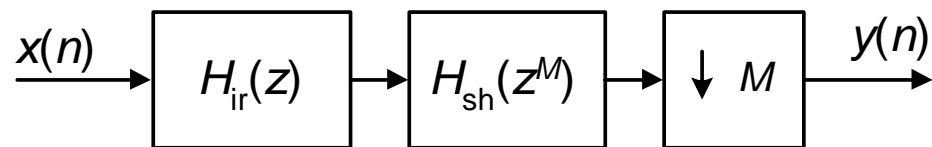
- ▶ **IFIR filters useful for signal sample rate change applications,**
 - **decimation or interpolation.**
- ▶ **Consider an IFIR filter followed by downsampling by integer M .**
 - **Operation ' $\downarrow M$ ' means discard all but every M th sample.**
- ▶ **Because $H_{sh}(z^M)$ and $H_{ir}(z)$ are linear, we can swap their order.**



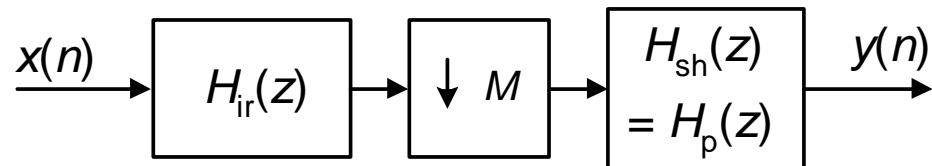
Decimation



- ▶ Here comes the good part.
- ▶ We can swap the order of the $H_{sh}(z^M)$ filter with the downsampler.
- ▶ Now, where every M -unit delay in $H_{sh}(z^M)$ is replaced by a unit delay.



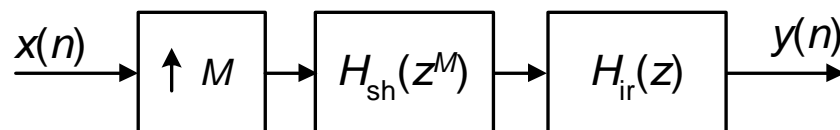
Decimation



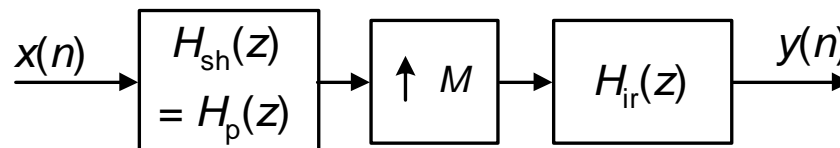
- ▶ This takes us back to using our original low-order prototype filter $H_p(z)$,
 - with its reduced signal data storage requirements. 😊
- ▶ Also, the $H_{ir}(z)$ and M downsampler combination can use polyphase filtering to reduce computational workload [1].

▶ Similarly, IFIR filters can be used for interpolation (upsampling).

- The upsampling (interpolation) operation ' $\uparrow M$ ' means insert $M-1$ zero-valued samples between each $x(n)$ sample.



Interpolation



- ▶ We swap the order of filter $H_{sh}(z^M)$ with the upsampler,
- ▶ Now every M -unit delay in $H_{sh}(z^M)$ is replaced by a unit delay.
- ▶ This takes us back to using our original low-order prototype filter $H_p(z)$,
 - with its reduced signal data storage requirements. 😊
- ▶ The M upsampler and $H_{ir}(z)$ combination can use polyphase filtering to reduce computational workload.

IFIR Filter Summary

- ▶ **We've introduced the structure and performance of IFIR filters.**
- ▶ **IFIR filters they can achieve significant computational workload reduction relative to traditional nonrecursive FIR filters,**
 - **reductions as large as 90%.**
- ▶ **IFIR filter implementation is a cascade of filters simple tapped-delay line FIR filters,**
 - **designed using readily-available nonrecursive FIR filter design software.**

- ▶ **More IFIR filter details,**
 - **math derivations**
 - **design guidelines, and**
 - **additional literature references are provided in:**

Reference [1]:

Understanding Digital Signal Processing,
2nd Ed., by R. Lyons, Prentice Hall, Upper
Saddle River, New Jersey, 2004

